Capacity Investments in a Stochastic Dynamic Game: Equilibrium Characterization

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Abstract

We study a two-period duopoly game with capacity accumulation under demand uncertainty, production and time-to-build constraints. These considerations are relevant in network industries such as electricity markets and hot spot markets.We characterize and compare open-loop, Markov perfect and closed-loop Nash equilibrium investments.

Key Words: Production Capacity Investment, Dynamic Game, Open-loop Equilibrium, Markov perfect Equilibrium, Closed-loop Equilibrium.

1 Introduction

In many industries capacity investments are made under uncertainty. Uncertainty may stem from the nature of production characteristics, demand, cost or macroeconomic conditions. Some uncertainties are industry specific and the degree of uncertainty may vary from industry to industry. Production capacity investments under uncertainty have been studied extensively in the literature. The recent studies revisit and extend the early contributions to incorporate different demand models and behavioral assumptions to study the new capital intensive markets including, e.g., restructured electric power generation, natural gas transportation, ethanol, and semiconductor (e.g., Chou et al. [1]). The main objectives of these articles are to provide insights for equilibrium investment behavior. However, the capacity competition over time, in which capacity is subject to a time-to-build constraint and firms face demand uncertainties over time, has not been adequately analyzed. (According to the empirical study by Koeva [2], time-to-build ranges from 13 to 86 months.) In particular, how firms would adjust their incremental capacity investments over time under different behavioral assumptions (precommitment versus no commitment, or open-loop versus Markov perfect or closed-loop) is an important question to be investigated. For example, in the electricity production industry competing power generation firms can invest incrementally in some technologies under demand uncertainty either using some precommitment policies or using some state-dependent policies.

We study a two-period duopoly game with capacity accumulation under demand uncertainty. Investment is not productive instantly, and there is a lag between investment and production. We characterize and compare open-loop, Markov perfect and closed-loop Nash equilibrium investments. There is a significant literature in *deterministic* dynamic games comparing Markov perfect and open-loop strategies, see e.g., Reynolds [3], Driskill and McCafferty [4], Long et al. [5] and Figuières [6]. Few papers adopt a stochastic dynamic game framework, but did not compare the different equilibrium results (see, e.g., Haurie and Zaccour [7], Genc et al. [8], Chevalier-Roignant et al. [9]). Besides game-theoretic analysis, there is a vast literature examining capacity investments using real options framework (e.g., Li and Wang [10]).

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The contribution of this paper is twofold. First, we characterize and compare optimal capacity investment strategies under the three equilibrium concepts (open-loop, closed-loop, and Markov perfect) in a dynamic game. Second, we examine a richer model that incorporates demand uncertainty, capacity and time-to-build constraints simultaneously. We show how equilibrium outcomes change as these constraints play a role in equilibrium predictions.

The plan of the rest of the article is as follows. Section 2 introduces the model, and Section 3 states some general results along with the key findings. Section 4 concludes the paper.

2 Model

To analyze the impact of the information structure on equilibrium decisions, we adopt the simplest possible parsimonious model, that is, a two-period duopoly game in which firms produce a homogeneous good. At time $t = 0$ for a given demand and capacity state vector, firms produce competitively and make capacity investments simultaneously and non-cooperatively under demand uncertainty. The stochastic process we consider is a random walk with two states- upstate and downstate. An investment made at time $t = 0$ will become productive in the following period. After demand uncertainty is resolved, firms make production decisions simultaneously and independently at time $t = 1$.

There are two firms $\{i, -i\}$, who compete over two periods $t = 0, 1$. In period 0, inverse demand is known to be $P_0(Q) = 1 - Q$, with Q the total output of the two firms. The stochastic inverse demand in period 1 is:

$$P_1(Q) = \begin{cases} 1 + \xi - Q & \text{with probability } p \\ 1 - \xi - Q & \text{with probability } 1 - p \end{cases} \tag{1}$$

The demand at initial period has two successors with $1u$, $1d$ denoting demand shifting up or down. Denote I_{i0} the investment in the production capacity for player i. Assuming away obsolescence and taking into account the one-period delay for investment to become productive, the capacity accumulation dynamics is given by

$$K_{i1} = K_{i0} + I_{i0}. \tag{2}$$

Each player must satisfy the production capacity constraint at each production node,

$$q_{it} \leq K_{it}, \qquad t = 0, 1. \tag{3}$$

We adopt a quadratic investment cost function and a linear production cost,

$$F_i(I_i) = 1/2f\, I_i^2, \qquad C_i(q_i) = cq_i,$$

where $f > 0$ and $0 < c < 1$. Also $0 \leq \xi < 1 - c$. Assuming profit maximization behavior, each player maximizes expected discounted payoff with a common discount factor $\delta \in (0, 1)$ subject to the above constraints.

What follows is the formal definition of information structures, S-adapted open-loop and perfect memory closed-loop information, which we will employ in equilibrium predictions.

Definition 1 *S-adapted open-loop information: At any time each player's information set includes the current calendar time, the current demand state, the distribution of future demand, and the initial values of capacity states.*

Definition 2 *Perfect memory closed-loop information: At any time each player's information set includes the current calendar time, the current states involving demand and capacity states, the distribution of future demand, and the history of the states.*

We use the term S-adapted (sample adapted) to reflect the fact that the game is stochastic and the demand distribution is modeled by event tree. S-adapted equilibrium strategies in an open loop equilibrium allow the decisions to be adapted to demand shock realization but not the capacity (investment) decision in the previous period. Both S-adapted open-loop equilibrium (in short OLNE) and the perfect memory closed-loop equilibrium (CLNE) are Nash equilibrium in investment and production strategies. The former is obtained under the S-adapted open-loop information structure, and the latter is obtained under the perfect memory closed-loop information structure. In between these information structures is the Markov perfect information in which a player conditions his decision on the value of the current state only. This is the feedback rule that considers the current stock of states, irrespective of initial conditions (see Basar and Olsder [11]).

3 Results

We report some general results pertaining to production and investment decisions at any time and demand state. We also show relationships between output and investment decisions. These results provide some valuable insights for the characterization of open-loop, Markov perfect and closed-loop equilibria.

Consider production decisions at any time. Because of the lag between investment and production, the investment decision is independent of quantity decision at the same period. Player i chooses production quantity by solving the following profit maximization problem,

$$\max P_t(Q) q_{it} - C_i(q_{it}),$$
$$s.t., \quad 0 \leq q_{it} \leq K_{it}.$$

The solution of the problem produces four equilibrium candidates: (i) The interior Cournot solutions, $q_{it} = (1 + \hat{\xi} - c)/3 = q_{-it}$; or (ii) The corner solutions, $q_{it} = K_{it}$, $q_{-it} = K_{-it}$; or (iii) The asymmetric solution with player i producing at full capacity, $q_{it} = K_{it}$ and the rival player $-i$ playing its best response strategy $q_{-it} = (1 + \hat{\xi} - c - K_{it})/2$, where $\hat{\xi} \in \{0, \xi, -\xi\}$; or (iv) The asymmetric solution with player i producing at the interior Cournot level, $q_{it} = (1 + \hat{\xi} - c - K_{-it})/2$ and the rival player $-i$ producing at the capacity, $q_{-it} = K_{-it}$. The following lemma shows that the asymmetric solution is ruled out in a symmetric game.

Lemma 1 *At any time and demand state, whenever capacities of the players are symmetric, Nash equilibrium outputs are unique and symmetric.*

All proofs are in the Appendix.

The next lemma shows that it can never occur that a player's output in downstate $1d$ exceeds his production in upstate $1u$. Note that this result is independent of production capacities.

Lemma 2 *In period 1, $q_{i1d} \leq q_{i1u}$.*

The following lemma states that if a player invests at period 0, then this player will produce at full capacity in upstate.

Lemma 3 *If $I_{i0} > 0$, then player i produces at capacity in upstate, $q_{i1u} = K_{i1}$.*

Consider the simplest possible setting of rivalry investment decisions, where demand is known with certainty ($\tilde{\xi} = 0$). We have the following result.

Proposition 1 *In the absence of uncertainty, open-loop, Markov perfect and closed-loop Nash equilibrium investments coincide.*

The result holds because the closed-loop, Markov perfect and open-loop Nash equilibrium state vectors at each stage coincide, and the rollback solution is identical to the forward solution. Alternatively, as the investment cost is sunk for the second period and the effect of investment is to provide an upper bound for the production level, the equilibria coincide. Note that this result holds for any given initial production capacities. Further, as one can expect, it can be easily shown that total industry investment is lower than the welfare-maximizing level.

Now consider stochastic demand ($\tilde{\xi} > 0$). The interior Cournot outputs will be $q_0^c = (1 - c)/3$, $q_u^c = (1 + \xi - c)/3$, and $q_d^c = (1 - \xi - c)/3$ for time zero, upstate and downstate demand, resp. Depending on the model parameter values, several interesting investment profiles arise:

$$\text{Case 1} \quad : \quad I_{i0} = 0 \text{ and } q_{i1d} \leq q_{i1u} \leq K_{i1},$$
$$\text{Case 2} \quad : \quad I_{i0} > 0 \text{ and } q_{i1d} < q_{i1u} = K_{i1},$$
$$\text{Case 3} \quad : \quad I_{i0} > 0 \text{ and } q_{i1d} = q_{i1u} = K_{i1}.$$

Case 1 occurs when the player's initial capacity is high. Strict inequalities will hold when initial capacity is greater than Cournot output in high demand ($K_{i0} > (1 + \xi - c)/3$). Case 2 occurs (Proposition 2) when the initial capacity K_{i0} is large so that the capacity constraints do not always bind, but also low enough that firms have an incentive to invest in capacity ($I_{i0} > 0$). Case 3 occurs (Proposition 3) when the initial capacity is low and total capacity is binding in both states in period 1.

Proposition 2 *Assume* $K_{i0} = K_{-i0} = K_0$, *and initial capacity is high such that* $k' < K_0 < q_u^c$, *where* $k' = q_d^c(1 + \delta p/f) - 2\delta p \xi/f$. *Then, for firms* $i, -i = 1, 2$, $i \neq -i$

1. *Symmetric open-loop (OL), Markov perfect (MP) and closed-loop (CL) Nash equilibrium investments are,*
$$I_{i0}^{OL} = \frac{\delta p \left(1 + \xi - c - 3K_{i0}\right)}{f + 3\delta p} = I_{i0}^{MP}, \quad I_{i0}^{CL} = \frac{\delta p \left(1 + \xi - c - 4K_{i0}\right)}{f + 4\delta p}.$$

2. *Equilibrium quantities at time 1 are given by*
$$
\begin{aligned}
q_{i1u}^{CL} &= K_{i1}^{CL} = \frac{f K_{i0} + \delta p \left(1 + \xi - c\right)}{f + 4\delta p}, \quad q_{i1d}^{CL} = \frac{1 - \xi - c}{3}, \\
q_{i1u}^{OL} &= K_{i1}^{OL} = \frac{f K_{i0} + \delta p \left(1 + \xi - c\right)}{f + 3\delta p} = q_{i1u}^{MP}, \quad q_{i1d}^{OL} = \frac{1 - \xi - c}{3} = q_{i1d}^{MP}.
\end{aligned}
$$

3. *Equilibrium profits compare*
$$\pi_i^{OL} = \pi_i^{MP} < \pi_i^{CL}.$$

4. *Asymmetric equilibrium in investment strategies is not possible.*

Note that k', the restriction on the lower bound of initial capacity, is chosen to restrict the equilibria so that equilibrium investment levels are comparable under the different information structures. Also, at the chosen level of k' we end up with Case 2 so that firms will invest and the investment will be fully utilized whenever the upstate demand unfolds. As shown in the proof of this proposition, there is a large lower bound of initial capacity in which investment strategies satisfy the properties in Case 2. However, to be able to compare the investment levels under OL and CL structures we make sure that these investment strategies are well-defined in the same parameters region. Therefore, we consider the relevant region of initial capacities (i.e., intersection of the lower bounds), which is the restricted region, so that investment expressions are comparable.

Proposition 3 *Assume* $K_{i0} = K_{-i0} = K_0$, *and initial capacity is low such that* $0 < K_0 < k''$, *where* $k'' = q_d^c - 2\delta p \xi/f$. *Then, for firms* $i, -i = 1, 2$, $i \neq -i$

1. *Symmetric open-loop, Markov perfect and closed-loop Nash equilibrium investments are,*
$$I_{i0}^{OL} = \frac{\delta \left(1 - \xi - c - 3K_{i0} + 2p\xi\right)}{f + 3\delta} = I_{i0}^{MP}, \quad I_{i0}^{CL} = \frac{\delta \left(1 - \xi - c - 4K_{i0} + 2p\xi\right)}{f + 4\delta}.$$

2. *Equilibrium production quantities at time 1 are*
$$
\begin{aligned}
q_{i1u}^{CL} &= q_{i1d}^{CL} = K_{i1}^{CL} = \frac{f K_{i0} + \delta \left(1 + \xi - c + 2p\xi\right)}{f + 4\delta}, \\
q_{i1u}^{OL} &= q_{i1d}^{OL} = q_{i1u}^{MP} = q_{i1d}^{MP} = K_{i1}^{OL} = \frac{f K_{i0} + \delta \left(1 + \xi - c + 2p\xi\right)}{f + 3\delta} = K_{i1}^{MP}.
\end{aligned}
$$

3. *Equilibrium profits compare*
$$\pi_i^{OL} = \pi_i^{MP} < \pi_i^{CL}.$$

4. *Asymmetric equilibrium in investment strategies is not possible.*

Proof. See the Appendix. □

Note that the upper bound of initial capacity k'' ensures that we are analyzing Case 3 and the equilibria under OL, MP and CL information structures are comparable.

Contrasting Propositions 2 and 3 with Proposition 1 confirms the known conclusion that under uncertainty the three information structures do not produce the same investments, see, e.g., Haurie et al. [12]. Further, a simple comparison of the investment strategies in Proposition 2, as well as in Proposition 3, shows that $I_{i0}^{OL} = I_{i0}^{MP} > I_{i0}^{CL}$, and, therefore, open-loop Nash equilibrium capacity per firm exceeds its closed-loop Nash equilibrium counterpart. Interestingly, open-loop equilibrium is subgame perfect, which stems from the binding capacity constraints. Under the closed-loop structure firms, a firm's output in the upstate is increasing in the rival firm's investment. This creates collusive-like behavior in which when a firm reduces its investment the rival also decreases its investment. Hence closed-loop investment levels will be lower than the open-loop counterparts. Under the OLNE a firm's period zero investment does not have any strategic value because it does not have an impact on its rival's period one output choice. Finally, under all information structures there is no room for an asymmetric equilibrium in investments strategies.

Propositions 2 and 3 show that investment is a function of demand probabilities p. It can be easily shown that investments are increasing in p. When $p = 0$ investments are zero in Proposition 2 . This implies that if upstate demand is not likely to unfold, which happens when the available capacities exceed the Cournot output in low demand, no investment occurs. For $p = 0$ investments are positive in Proposition 3 since they will be used in low demand state as well. At the initial period investments are made to benefit any future demand state -low or high- because firms start with installed capacities lower than the Cournot output in down state.

It is easy to check that if both players do not invest in capacity (which happens when initial capacities are enough to cover the upstate demand), then open-loop, Markov perfect and closed-loop outputs coincide at each node. Consequently, individual profits are the same under all information structures.

Proposition 4 *Assume asymmetric initial capacities, $K_{i0} \neq K_{-i0}$, and the capacities satisfy $k < K_{i0} < q_u^c < K_{-i0}$, where $k = q_d^c - \delta p \xi / f$. Then, the asymmetric OLNE, MPE and CLNE investments are given by*

$$I_{i0}^{OL} = \frac{\delta p[1 + \xi - c - 3K_{i0}]}{2f + 3\delta p}, \quad I_{i0}^{MP} = I_{i0}^{CL} = \frac{\delta p[1 + \xi - c - 2K_{i0}]}{2f + 2\delta p}, \quad I_{-i0}^{OL} = 0 = I_{-i0}^{CL} = I_{-i0}^{MP}$$

Further,

$$\pi_i^{OL} < \pi_i^{CL} = \pi_i^{MP},$$
$$\pi_{-i}^{OL} > \pi_{-i}^{CL} = \pi_{-i}^{MP}.$$

In this proposition, duopolists start with different initial capacities and in equilibrium one duopolist makes positive investment and the other does not. Facing a rival firm with large capacity, a player will invest less and realize lower profit in OLNE than in MPE (or CLNE). The MPE capacity for firm i exceeds its OLNE. Note that, player i produces at full capacity in the upstate and player $-i$ produces less than his capacity.

In this asymmetric game, this proposition only deals with the case in which one player invests and the other does not invest in equilibrium. The investing firm's capacity will be binding in the upstate alone. There are other possible asymmetric equilibria. For example, we could have characterized the equilibrium in which the investing firm's capacity would be binding in both states. However, the equilibrium characterization for that case will be qualitatively the same as with Proposition 4. There are some other equilibrium types, but for the sake of briefness we omit their characterizations.

4   Concluding Remarks

Although our setting is on purpose simple, there are several key notions behind the results. First one is the uncertainty, which is affecting the number of states in each time. In the certainty case (Proposition 1), the equilibrium investments coincide for all information structures. However, allowing uncertainty (in Proposition 2-4) generates different market outcomes and equilibrium ranking. As the uncertainty (represented by ξ) increases, product demand increases in the upstate, which creates incentives to invest in the earlier stage.

Firms will invest and produce more at high prices and hence the expected profit will increase. This results holds true under each equilibrium type.

The second key component is the capacity constraints. Depending on whether capacity constraints are binding or not equilibrium outcomes multiply: we obtain equilibrium prices, investments and outputs for interior and corner solutions, which covers the entire range of equilibria. In Proposition 2 capacity constraints were binding only in the high demand scenario, in Proposition 3 they were binding in both high and low demand situations. The status of the capacity constraints changes the profitability, market prices, and investment levels for a given equilibrium concept, but not the equilibrium ranking across the information structures.

Finally, the third driving force is the time-to-build constraint, which is a realistic feature that all firms face. Time-to-build constraint creates "here-and-now" investment decision which produces different equilibrium predictions than the instantaneous investment (no lead time) problems. For example, Reynolds [3] finds that MPE investments always exceed the OLNE investments when there is no lag between investment and production. However, we show in Propositions 2-4 that time-to-build leads to a different ranking of investment profiles.

An extension of the investment model to include more than two periods does not cause any conceptual difficulty. However, the full characterization of the different equilibria and their comparison will not be feasible analytically as the number of possibilities increases exponentially. This comparison may still be carried out numerically, that is, on examples of interest with specific parameter values.

Acknowledgements

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APPENDIX

Proof of Lemma 1

Cases emerge depending on whether the capacity is greater or lower than the unconstrained Cournot equilibrium outputs. If $K \geq (1 + \xi - c)/3$ and player i plays $q_i = K$ then the best response of player $-i$ is $q_{-i} = (1 + \xi - c - K)/2$ by the profit maximization. In that case, $P(Q) = (1 + \xi + c - K)/2$, and the profit of player i is $\pi_i = (1 + \xi - c - K)K/2$. However, player i can do better, namely its best response to player $-i$ strategy q_{-i} is $q_i^* = (1 + \xi - c + K)/4$. Then, $P^*(Q) = (1 + \xi + 3c + K)/2$, and player i's profit is $\pi_i^* = ((1 + \xi - c + K)/4)^2$. Then, clearly, $\pi_i^* \geq \pi_i$ if and only if $(1 + \xi - c - 3K)^2 \geq 0$, but this inequality holds because the production constraint must satisfy $q_i^* = (1 + \xi - c + K)/4 \leq K$. Hence, asymmetric outcomes are not part of the equilibrium. If $K < (1 + \xi - c)/3$ and player i plays K then the best response of player $-i$ is K, that is if the capacity K is lower than the symmetric Cournot level then the capacity constraints must be binding. If capacity K is greater than the symmetric Cournot outputs then the solution is the interior one. If K is equal to the Cournot outputs then the interior solution coincides with the corner solution. Therefore depending on the capacity level, the equilibrium will be unique.

Alternatively, the proof can directly be derived from the general solution, where the asymmetric solution for symmetric capacities immediately leads to a contradiction.

Proof of Lemma 2

There are three possible cases. Case 1: For a player the output in the upstate is greater than the output in the downstate and for the other player the reverse applies. Case 2: Each player produces higher output in the downstate. Case 3, which is the claim of the Lemma, that is, each player produces higher or equal outputs in the upstate than in the downstate. We will show that first two cases are not possible.

The Lagrangian of the profit maximization problem is

$$L_l = P_s(Q) q_{ls} - C_l(q_{ls}) + \lambda_{ls}(K_l - q_{ls}).$$

The first order conditions (FOC) lead to

$$P_s(Q) + P_s'(Q) q_{ls} - C_l'(q_{ls}) - \lambda_{ls} = 0, \quad s = 1u, \, 1d, \, l = i, \, -i.$$

Case 1: Assume the supposition $q_{i1u} > q_{i1d}$ and $q_{-i1u} < q_{-i1d}$. Then the first inequality implies $\lambda_{i1d} = 0 \leq \lambda_{i1u}$. Linearity of the inverse demand curve and the convexity of the cost function together with $q_{i1u} > q_{i1d}$ and $\lambda_{i1d} = 0 \leq \lambda_{i1u}$ imply that the last 3 terms in the LHS of the FOC are more negative for the upstate than the downstate. So, by FOC we must have $P_{1u}(Q) > P_{1d}(Q)$. On the other hand, the inequality $q_{-i1u} < q_{-i1d}$ implies that $\lambda_{-i1d} \geq \lambda_{-i1u} = 0$. Linearity of the inverse demand curve, the convexity of the cost function along with the inequalities $q_{-i1u} < q_{-i1d}$ and $\lambda_{-i1d} \geq \lambda_{-i1u} = 0$, lead to $P_{1u}(Q) < P_{1d}(Q)$ by the FOC. These inequalities of price ranking is obviously a contradiction. Therefore, it cannot happen that one player produces higher output in the one state and the other produces higher output in the another state.

Case 2: Assume the supposition $q_{l1u} < q_{l1d}$, $l = i, -i$. Then clearly $Q_{1u} < Q_{1d}$ must hold. Also $\lambda_{l1d} \geq \lambda_{l1u} = 0$ satisfies. By the properties of inverse demand and cost function and the supposition, the FOC results in

$$
\begin{aligned}
P_{1u}(Q) &= -P_{1u}'(Q) q_{l1u} + C_l'(q_{l1u}) \\
&< -P_{1d}'(Q) q_{l1d} + C_l'(q_{l1d}) \leq P_{1d}(Q),
\end{aligned}
$$

which implies that, due to the linearity of the inverse demand, $Q_{1u} > Q_{1d}$ which contradicts to $Q_{1u} < Q_{1d}$. Therefore, it cannot happen that both players produce higher outputs in the downstate.

Therefore, the result that firms do not produce lower outputs in the upstate than the downstate follows.

Proof of Lemma 3

Consider the optimization problem of player i in period 0, with the two successor states $1u$ and $1d$. It is straightforward to verify that the OL Nash equilibrium (OLNE) conditions include

$$\frac{\partial \pi_i}{\partial I_{i0}} = -f I_{i0} + \lambda_{i1u} + \lambda_{i1d} = 0,$$

$$\frac{\partial \pi_i}{\partial q_{i1u}} = \delta p \left[1 + \xi - 2q_{i1u} - q_{-i1u} - c\right] - \lambda_{i1u} = 0,$$

$$\lambda_{i1u} \geq 0, \qquad K_{i0} + I_{i0} - q_{i1u} \geq 0,$$

$$0 = \lambda_{i1u} \left[K_{i0} + I_{i0} - q_{i1u}\right],$$

$$\frac{\partial \pi_i}{\partial q_{i1d}} = \delta \left(1 - p\right) \left[1 - \xi - 2q_{i1d} - q_{-i1d} - c\right] - \lambda_{i1d} = 0,$$

$$\lambda_{i1d} \geq 0, \qquad K_{i0} + I_{i0} - q_{i1d} \geq 0,$$

$$0 = \lambda_{i1d} \left[K_{i0} + I_{i0} - q_{i1d}\right].$$

For $I_{i0} > 0$, we have $\lambda_{i1u} + \lambda_{i1d} > 0$. We have the following possibilities

$$\lambda_{i1u} > 0 \quad \text{and} \quad \lambda_{i1d} > 0,$$

$$\lambda_{i1u} > 0 \quad \text{and} \quad \lambda_{i1d} = 0,$$

$$\lambda_{i1u} = 0 \quad \text{and} \quad \lambda_{i1d} > 0.$$

The last possibility is excluded because only the condition $q_{i1u} \leq K_{i1} = q_{i1d}$ implies this possibility. But this condition contradicts to the Lemma 2 which proves that $q_{i1u} \geq q_{i1d}$. Hence, in all events we have $\lambda_{i1u} > 0$, and from complementarity conditions we must have $q_{i1u} = K_{i0} + I_{i0}$.

The proof of the result for the Markov perfect and closed-loop Nash equilibria are also similar. The structure of the proof is available in the following propositions.

Proof of Proposition 1

In this deterministic case, there is only one state in each period and therefore there is no need to distinguish between periods and states. Consider first the open-loop case. Player i maximizes

$$L_i = q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - f\, I_{i0}^2/2 + \delta[q_{i1}(1 - q_{i1} - q_{-i1}) - cq_{i1}]$$
$$+ \lambda_{i0}(K_{i0} - q_{i0}) + \lambda_{i1}(K_{i0} + I_{i0} - q_{i1}).$$

At time 0, the first order necessary conditions for production decisions (that are irrelevant of investment decisions) might yield several possibilities due to capacity constraints. It might produce interior Cournot solution: $q_{i0} = (1 - c)/3$ implying $\lambda_{i0} = 0$. Or, it might lead to one interior one corner solution: $q_{i0} = (1 - K_{-i0} - c)/2$ and $q_{-i0} = K_{-i0}$, yielding $\lambda_{i0} = 0$ and $\lambda_{-i0} > 0$. Or, both players are at the capacity: $q_{l0} = K_{l0}$, $l = i, -i$ implying $\lambda_{i0} \geq 0$ and $\lambda_{-i0} \geq 0$.

At time 1, the production quantities are the same as the ones above, except the state variable at that period might change with the possible capacity expansion made in earlier period. The optimum investment must solve the first order necessary conditions, which imply $I_{i0} = \lambda_{i1}/f$. Assuming positive investments by both firms means $\lambda_{i1} > 0$, which in turn implies, $K_{i1} + I_{i0} = q_{i1}$. The derivative of the objective function with respect to q_{i1} results in $\lambda_{i1} = \delta[1 - 2q_{i1} - q_{-i1} - c]$. Plugging this into the investment expression yields

$$f I_{i0} = \delta[1 - 2(K_{i0} + I_{i0}) - (K_{-i0} + I_{-i0}) - c],$$

The OLNE investment will satisfy this equality.

To characterize the Markov perfect equilibrium (MPE) investment levels we solve the problem backwards and start from the final stage. At time 1, the value function is

$$v_{i1} = q_{i1}(1 - q_{i1} - q_{-i1}) - cq_{i1} + \lambda_{i1}(K_{i0} + I_{i0} - q_{i1}).$$

The complementarity condition is, $\lambda_{i1}(K_{i0} + I_{i0} - q_{i1}) = 0$. Assuming that $\lambda_{i1} > 0$, we obtain the corner solution $q_{i1} = K_{i0} + I_{i0}$. Next we plug this expression into the value function and write the value function at time 0:

$$v_{i0} = q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - fI_{i0}^2/2 + \delta v_{i1}(I_{i0}) + \lambda_{i0}(K_{i0} - q_{i0}).$$

Taking the derivative with respect to the investment results in, assuming positive investments by both firms,

$$fI_{i0} = \delta[1 - 2(K_{i0} + I_{i0}) - (K_{-i0} + I_{-i0}) - c].$$

Clearly this expression is the same as the one obtained for OLNE. The computation of the closed-loop Nash equilibrium (CLNE) will also be same as the Markov perfect equilibrium in this certainty setting. Hence, investment levels coincide under all equilibrium concepts.

Proof of Proposition 2

First we characterize *open-loop Nash equilibrium* investments. We write the objective function to be maximized by firms $i = 1, 2$,

$$
\begin{aligned}
z_{i0} &= q_{i0}(1 - q_{i0} - q_{-i0}) - cq_{i0} - f\,I_{i0}^2/2 + \delta p[q_{i1u}(1 + \xi - q_{i1u} - q_{-i1u}) - cq_{i1u}] \\
&\quad + \delta(1 - p)[q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}] \\
&\quad + \lambda_{i0}(K_0 - q_{i0}) + \lambda_{i1u}(K_0 + I_{i0} - q_{i1u}) + \lambda_{i1d}(K_0 + I_{i0} - q_{i1d}).
\end{aligned}
$$

Under the assumption $k' < K_0 < q_u^c$, where the lower bound of initial capacity k' derived below, the initial capacity is low and firms undertake investment to increase the production capacity so as to meet the future demand. Due to Lemma 2 the production constraint will bind in the upstate, but the total capacity will be higher than the interior output in the downstate, and hence $\lambda_{i1u} > \lambda_{i1d} = 0$ will hold. It follows that $q_{i1u} = K_{i1u}$, $q_{i1d} = (1 - c - \xi)/3$. In period 1, capacity constraints only bind when demand is high, and the investment has an impact in the high demand state.

Taking the derivative of the above objective function (z_{i0}) with respect to the investment will yield to $I_{i0} = (\lambda_{i1u} + \lambda_{i1d})/f$, and the multipliers are obtained by solving $\frac{\partial z_{i0}}{\partial q_{i1u}} = 0$,

$$\lambda_{i1u} = \delta p[1 + \xi - c - 2q_{i1u} - q_{-i1u}] = \delta p[1 + \xi - c - 3(K_0 + I_{i0})],$$

and $\lambda_{i1d} = 0$. Then, the OLNE strategy as a function of the model parameters is

$$I_{i0}^{OL} = \frac{\delta p[1 + \xi - c - 3K_0]}{f + 3\delta p}, \quad i = 1, 2.$$

Next we characterize *Markov perfect equilibrium* investments. Under the assumption $k' < K_0 < q_u^c$, investment only benefits in the upstate demand. At time 1 state $1u$ player i maximizes

$$v_{iu} = [q_{i1u}(1 + \xi - q_{i1u} - q_{-i1u}) - cq_{i1u}] + \lambda_{i1u}(K_{i1u} - q_{i1u}),$$

where $K_{i1u} = I_{i0} + K_0$. The optimal output will satisfy $q_{i1u} = K_{i1u}$ because of the assumption. That is, at time 1 optimality conditions are $q_{i1u} = I_{i0} + K_0$ and $q_{-i1u} = I_{-i0} + K_0$.

At time 1 on state $1d$ player i maximizes

$$v_{id} = [q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}] + \lambda_{i1d}(K_{i1d} - q_{i1d}).$$

The optimum output will satisfy $q_{i1d} < K_{id}$ because of the assumption, where $K_{i1d} = K_{i1u}$ as up and down states share the same root/history.

At initial node, player i maximizes

$$
\begin{aligned}
v_{i0} &= q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - f\,I_{i0}^2/2 + \delta p w_{iu}(K_{i1u}, K_{-i1u}) + \delta(1 - p)w_{id}(.) \\
&\quad + \lambda_{i0}(K_0 - q_{i0}),
\end{aligned}
$$

where

$$w_{iu}(K_{i1u}, K_{-i1u}) = [(I_{i0} + K_{i0})(1 + \xi - I_{i0} - K_0 - I_{-i0} - K_0 - c)]$$

9

is the profit for player i at state $1u$ in period 1 when it has capacity of $K_{i1u} = I_{i0} + K_0$ and the rival has the capacity of $K_{-iu} = I_{-i0} + K_0$. Also

$$w_{id}(.) = q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}$$

is the profit for player i at state $1d$ in period 1. The optimal investment must satisfy the first order condition

$$-fI_{i0} + p\delta \frac{\partial w_{iu}}{\partial K_{i1u}} \frac{\partial K_{i1u}}{\partial I_{i0}} = 0,$$

or

$$-fI_{i0} + p\delta[1 + \xi - c - q_{-i1u} - 2K_{i1u} - K_{i1u} \frac{\partial K_{-i1u}}{\partial I_{i0}}] \quad = \quad 0, \tag{4}$$

$$-fI_{-i0} + p\delta[1 + \xi - c - q_{i1u} - 2K_{-i1u} - K_{-i1u} \frac{\partial K_{i1u}}{\partial I_{-i0}}] \quad = \quad 0, \tag{5}$$

for players i and $-i$ respectively.

Plugging the outputs into (4) and (5) and driving $\frac{\partial K_{-i1u}}{\partial I_{i0}} = 0 = \frac{\partial K_{i1u}}{\partial I_{-i0}}$ at time 1 and solving (4) and (5) simultaneously we have

$$I_{i0}^{MP} = \frac{\delta p[1 + \xi - c - 3K_0]}{f + 3\delta p}, \quad i = 1, 2.$$

The equilibrium production quantities at time 1 will satisfy $q_{1u} = (K_{i1u}, K_{-i1u})$ at the upstate demand, and $q_{1d} = ((1 - \xi - c)/3, (1 - \xi - c)/3)$ at the downstate demand. Since Markov perfect investment coincides with the open-loop one, they will produce the identical outputs and profits.

Next we characterize *closed-loop Nash equilibrium (with memory)* investments. Players still solve the problem backwards as they do in the Markov perfect equilibrium. The only difference is the information structure. Namely, players remember the past decisions and take them into account while making the current decisions.

Under the assumption $k' < K_0 < q_u^c$, investment only benefits the upstate demand. At time 1 state $1u$ player i maximizes

$$v_{iu} = [q_{i1u}(1 + \xi - q_{i1u} - q_{-i1u}) - cq_{i1u}] + \lambda_{i1u}(K_{i1u} - q_{i1u}),$$

where $K_{i1u} = I_{i0} + K_0$. The optimum output will satisfy $q_{i1u} = K_{i1u}$ because of the assumption.

At time 1 at state d player i maximizes

$$v_{id} = [q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}] + \lambda_{i1d}(K_{i1u} - q_{i1d}).$$

The optimum output will satisfy $q_{i1d} < K_{i1d}$, where $K_{i1d} = K_{i1u}$, because of the assumption.

At initial node, player i maximizes

$$\begin{aligned} v_{i0} &= q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - f I_{i0}^2/2 + \delta p w_{iu}(K_{i1u}, K_{-i1u}) + \delta(1-p)w_{id}(.) \\ &\quad + \lambda_{i0}(K_0 - q_{i0}), \end{aligned}$$

where $w_{iu}(K_{i1u}, K_{-i1u})$ is the profit for player i at state $1u$ in period 1 when it has capacity of $K_{i1u} = I_{i0} + K_0$ and the rival has the capacity of $K_{-iu} = I_{-i0} + K_0$. Also $w_{id}(.) = q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}$ is the profit for player i at state $1d$ in period 1. The optimal investment must satisfy

$$-fI_{i0} + p\delta \frac{\partial w_{iu}}{\partial K_{iu}} \frac{\partial K_{iu}}{\partial I_{i0}} = 0,$$

or

$$-fI_{i0} + p\delta[1 + \xi - c - q_{-i1u} - 2K_{iu} - K_{iu} \frac{\partial K_{-i1u}}{\partial I_{i0}}] \quad = \quad 0, \tag{6}$$

$$-fI_{-i0} + p\delta[1 + \xi - c - q_{i1u} - 2K_{-iu} - K_{-iu} \frac{\partial K_{i1u}}{\partial I_{-i0}}] \quad = \quad 0, \tag{7}$$

for players i and $-i$ respectively.

At time 1 optimality conditions are $q_{i1u} = I_{i0} + K_0$ and $q_{-i1u} = I_{-i0} + K_0$. At time 0 both players can derive the optimality conditions (6) and (7). They observe that these expressions are symmetric and at period 1 they will produce at the capacity. Therefore, the same investment level must solve (6) and (7) simultaneously. Hence, period 1 outputs must be identical. That is, $q_{i1u} = I_{i0} + K_0 = q_{-i1u} = I_{-i0} + K_0$. And then, $\frac{\partial K_{-i1u}}{\partial I_{i0}} = 1 = \frac{\partial K_{i1u}}{\partial I_{-i0}}$ must satisfy. Given these conditions, we solve (6) and (7) to obtain

$$I_{i0}^{CL} = \frac{\delta p[1 + \xi - c - 4K_0]}{f + 4\delta p}, \quad i = 1, 2.$$

The equilibrium production quantities at time 1 will satisfy $q_{1u} = (K_{i1u}, K_{-i1u})$ at the upstate demand, and $q_{1d} = ((1 - \xi - c)/3, (1 - \xi - c)/3)$ at the downstate demand.

Mathematically, the difference between CLNE and MPE investment levels is that under the CLNE $\frac{\partial K_{-i1u}}{\partial I_{i0}} = 1 = \frac{\partial K_{i1u}}{\partial I_{-i0}}$ must satisfy, however under the MPE $\frac{\partial K_{-i1u}}{\partial I_{i0}} = 0 = \frac{\partial K_{i1u}}{\partial I_{-i0}}$ holds at time 1 which generates the difference between the equilibrium predictions.

Next we obtain the lower bound of initial capacity, k', that entails non-binding capacity at the downstate demand by solving $K_0 + I_{i0} > q_d^c$. When we insert I_{i0}^{OL} into this inequality, we obtain that $K_0 > q_d^c - 2\delta\xi p/f$. When we insert I_{i0}^{CL} into this inequality we obtain $K_0 > q_d^c(1 + \delta p/f) - 2\delta\xi p/f$. To make the investment levels comparable in these regions we take the maximum of these bounds; $max[q_d^c - 2\delta\xi p/f, \ q_d^c(1 + \delta p/f) - 2\delta\xi p/f] = k'$.

We now show that $\pi_i^{CL} > \pi_i^{OL}$. The CLNE and OLNE profits at initial node and node d in period 1 are clearly the same. Therefore, we need to compare the profits at node u in period 1. The difference in profits is given by

$$\pi_i^{OL} - \pi_i^{CL} = A + B,$$

where, dropping the player index,

$$
\begin{aligned}
A &= -f((I^{OL})^2 - (I^{CL})^2)/2 = -f(I^{OL} - I^{CL})(I^{OL} + I^{CL})/2, \\
B &= \delta p[(K_0 + I^{OL})(1 + \xi - 2(K_0 + I^{OL}) - c) \\
&\quad -(K_0 + I^{CL})(1 + \xi - 2(K_0 + I^{CL}) - c)].
\end{aligned}
$$

Because $I^{OL} > I^{CL}$, A is negative. If the sign of B is negative, then we are done. Otherwise, we need to determine the sign of $|A| - B$. We have

$$
\begin{aligned}
B &= \delta p[(K_0 + I^{OL})(1 + \xi - 2(K_0 + I^{OL}) - c) \\
&\quad -(K_0 + I^{CL})(1 + \xi - 2(K_0 + I^{CL}) - c)] \\
&= \delta p[-2K_0 I^{OL} + 2K_0 I^{CL} + I^{OL}(1 + \xi - 2(K_0 + I^{OL}) - c) \\
&\quad -I^{CL}(1 + \xi - 2(K_0 + I^{CL}) - c)] \\
&= \delta p[2K_0(I^{CL} - I^{OL}) + (1 + \xi - 2K_0 - c)(I^{OL} - I^{CL}) - 2(I^{OL})^2 + 2(I^{CL})^2] \\
&= \delta p[(I^{OL} - I^{CL})(1 + \xi - 4K_0 - c) - 2(I^{OL} - I^{CL})(I^{OL} + I^{CL})] \\
&= \delta p[(I^{OL} - I^{CL})(1 + \xi - c - 4K_0 - 2(I^{OL} + I^{CL}))].
\end{aligned}
$$

In the expression $\pi_i^{OL} - \pi_i^{CL} = A + B$, we will show that $|A| > B$. Indeed,

$$
\begin{aligned}
|A| - B &= (I^{OL} - I^{CL})[\frac{f(I^{OL} + I^{CL})}{2} - \delta p(1 + \xi - c - 4K_0 - 2(I^{OL} + I^{CL}))] \\
&= (I^{OL} - I^{CL})[(I^{OL} + I^{CL})(2\delta p + f/2) - \delta p(1 + \xi - c - 4K_0)] \\
&= (I^{OL} - I^{CL})[(I^{OL} + I^{CL})(2\delta p + f/2) - I^{CL}(f + 4\delta p)] \\
&= (I^{OL} - I^{CL})[(I^{OL}(2\delta p + f/2) - I^{CL}(2\delta p + f/2)] \\
&= (I^{OL} - I^{CL})^2(2\delta p + f/2) > 0.
\end{aligned}
$$

Hence, $\pi_i^{OL} - \pi_i^{CL} < 0$.

Next we show that asymmetric equilibrium in investment strategies is not possible. That is, since $K_{i0} = K_0 = K_{-i0}$ and investment is positive $I_{i0}^{CL} = I_{-i0}^{CL}$, and $I_{i0}^{OL} = I_{-i0}^{OL}$ must hold. To see this in the OLNE we look at the investment expression, $I_{i0} = \lambda_{i1u}/f$, where

$$\lambda_{i1u} = \delta p[1 + \xi - 2q_{i1u} - q_{-i1u}] = \delta p[1 + \xi - c - 2(K_0 + I_{i0}) - (K_0 + I_{-i0})].$$

Then, we will have

$$
\begin{aligned}
I_{i0}^{OL} &= \frac{\delta p[1 + \xi - c - 3K_0 - 2I_{i0}^{OL} - I_{-i0}^{OL}]}{f}, \\
I_{-i0}^{OL} &= \frac{\delta p[1 + \xi - c - 3K_0 - 2I_{-i0}^{OL} - I_{i0}^{OL}]}{f},
\end{aligned}
$$

which are clearly symmetric expressions and the only solution is $I_{i0}^{OL} = I_{-i0}^{OL}$.

In the CLNE at initial node player i maximizes

$$v_{i0} = v - f\, I_{i0}^2/2 + \delta p[(K_0 + I_{i0})(1 + \xi - c - 2K_0 - I_{i0} - I_{-i0})],$$

where v is the portion of the profit not involving the investment term. Taking the derivative of this expression with respect to I_{i0} and equating it to zero yield

$$I_{i0}^{CL} = \frac{\delta p[1 + \xi - c - 3K_0 - I_{-i0}^{CL}]}{f + 2\delta p}.$$

Similarly, for player j we obtain

$$I_{-i0}^{CL} = \frac{\delta p[1 + d - c - 3K_0 - I_{i0}^{CL}]}{f + 2\delta p}.$$

Clearly these best response functions admit a unique symmetric solution. Hence $I_{i0}^{CL} = I_{-i0}^{CL}$.

Proof of Proposition 3

The proof is similar to the proof of Proposition 2 and is omitted. But we will derive the lower bound of initial capacity, k'', that entails binding capacity at both upstate demand and the downstate demand by solving $K_{i0} + I_{i0} < q_d^c$. When we insert I_{i0}^{OL} into this inequality, we obtain that $K_{i0} < q_d^c - 2\delta\xi p/f$. When we insert I_{i0}^{CL} into this inequality we obtain $K_{i0} < q_d^c(1 + \delta/f) - 2\delta\xi p/f$. To make the investment levels comparable in these regions we take the minimum of these bounds;

$$\min[q_d^c - 2\delta\xi p/f, \; q_d^c(1 + \delta/f) - 2\delta\xi p/f] = k''.$$

Proof of Proposition 4

We write the objective function to be maximized by firm i:

$$
\begin{aligned}
v_{i0} &= q_{i0}(1 - q_{i0} - q_{-i0}) - cq_{i0} - f\, I_{i0}^2/2 + \delta p[q_{i1u}(1 + \xi - q_{i1u} - q_{-i1u}) - cq_{i1u}] + \\
&\quad \delta(1 - p)[q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}] \\
&\quad + \lambda_{i0}(K_{i0} - q_{i0}) + \lambda_{i1u}(K_{i0} + I_{i0} - q_{i1u}) + \lambda_{i1d}(K_{i0} + I_{i0} - q_{i1d}).
\end{aligned}
$$

Under the assumption $k < K_{i0} < q_u^c < K_{-i0}$, in which the expression for k is derived below, the initial capacity for player i is low and it has to invest given the larger market share of his competitor $-i$. The addition of new investment will facilitate 100% capacity utilization to the firm i once high demand scenario unfolds. The firm i will use some of its investment in the production process if it faces low demand in the market. However, due to the abundance of its initial production capacity the rival firm$-i$ will opt out of investing.

First we characterize open-loop investment strategy. We optimize the above objective function and obtain that $I_{i0} = \lambda_{i1}/f$, and $\lambda_{i1} = \delta p[1 + \xi - c - 2q_{i1u} - q_{-i1u}]$, where $q_{i1u} = K_{i0} + I_{i0}$, and $q_{-i1u} = (1 + \xi - c - K_{i0} - I_{i0})/2$. Then the OLNE investment will be equal to $I_{i0}^{OL} = \dfrac{p\delta[1 + \xi - c - 3K_{i0}]}{2f + 3p\delta}$.

Next we characterize Markov perfect equilibrium investments. At the upstate demand $q_{i1u} = K_{i0} + I_{i0}$, and $q_{-i1u} = (1 + \xi - c - K_{i0} - I_{i0})/2$ will hold. At the downstate demand, we have $q_{i1d} = (1 - \xi - c)/3 = q_{-i1d}$. We plug these expressions into the respective objective function and maximize with respect to I_{i0} for firm i. The Markov perfect equilibrium investment strategy will be equal to $I_{i0}^{MP} = \dfrac{p\delta[1 + \xi - c - 2K_{i0}]}{2f + 2p\delta}$. The closed-loop equilibrium investment level will be identical to the Markov perfect investment, because $\frac{\partial q_{-i1u}}{\partial I_{i0}} = -1/2$, $\frac{\partial q_{i1u}}{\partial I_{i0}} = 1$, and $\frac{\partial q_{i1u}}{\partial I_{-i0}} = 0$. Clearly, $I_{i0}^{CL} = I_{i0}^{MP} > I_{i0}^{OL}$ holds.

We derive the lower bound of initial capacity, k, that entails binding capacity at the upstate demand for firm i by solving $K_{i0} + I_{i0} > q_d^c$. When we insert I_{i0}^{OL} into this inequality, we obtain that $K_{i0} > q_d^c - \delta \xi p/f$. When we insert I_{i0}^{CL} into this inequality we obtain $K_{i0} > q_d^c(1 - \delta p/2f) - \delta \xi p/f$. To make the investment levels comparable for firm i under both equilibrium concepts, we take the maximum of these bounds;

$$k = \max[q_d^c - \delta \xi p/f, \ q_d^c(1 - \delta p/2f) - \delta \xi p/f] = q_d^c - \delta \xi p/f.$$

We now show that $\pi_i^{CL} > \pi_i^{OL}$. We have

$$
\begin{aligned}
\pi_i^{CL} &= \Pi - f\left(I_{i0}^{CL}\right)^2/2 + \delta p(K_{i0} + I_{i0}^{CL})(1 + \xi - (K_{i0} + I_{i0}^{CL}) \\
&\quad -\delta p(1 + \xi - c - K_{i0} - I_{i0}^{CL})/2 - c), \\
\pi_i^{OL} &= \Pi - f\left(I_{i0}^{OL}\right)^2/2 + \delta p(K_{i0} + I_{i0}^{OL})(1 + \xi - (K_{i0} + I_{i0}^{OL}) \\
&\quad -\delta p(1 + \xi - c - K_{i0} - I_{i0}^{OL})/2 - c)],
\end{aligned}
$$

where Π is the profit term involving initial node and node d in period 1. The profit difference is thus given by

$$
\begin{aligned}
\pi_i^{OL} - \pi_i^{CL} &= -f((I^{OL})^2 - (I^{CL})^2)/2 + \delta p[(K_0 + I^{OL})(1 + \xi - (K_0 + I^{OL}) - c) \\
&\quad -(K_0 + I^{CL})(1 + \xi - (K_0 + I^{CL}) - c)]/2.
\end{aligned}
$$

Let

$$
\begin{aligned}
A &= -f((I^{OL})^2 - (I^{CL})^2)/2, \\
B &= \frac{\delta p}{2}[(K_0 + I^{OL})(1 + \xi - (K_0 + I^{OL}) - c) - (K_0 + I^{CL})(1 + \xi - (K_0 + I^{CL}) - c)].
\end{aligned}
$$

A is positive because $(I^{OL} - I^{CL})(I^{OL} + I^{CL}) < 0$ because $I^{OL} < I^{CL}$. It is easy to check that B reduces to

$$B = \delta p[(I^{OL} - I^{CL})(1 + \xi - c - 2K_0 - (I^{OL} + I^{CL}))].$$

Now, note that

$$
\begin{aligned}
\pi_i^{OL} - \pi_i^{CL} &= A + B, \\
&= (I^{MP} - I^{OL})(I^{OL} - I^{CL})(\delta p/2 + f/2),
\end{aligned}
$$

which is negative, and hence $\pi_i^{OL} < \pi_i^{CL}$.

We next show that $\pi_{-i}^{CL} < \pi_{-i}^{OL}$ for player $-i$. Similar to the profit difference for player i, the profit difference for player $-i$ under both equilibria boils downs to

$$\pi_{-i}^{OL} - \pi_{-i}^{CL} = \delta p(I^{CL} - I^{OL})(2 + 2\xi - 2c - 2K_0 + I^{CL} - I^{OL})/4.$$

Note that the investment levels I^{CL}, I^{OL} are the investments made by player i. The difference is positive because both the first term and the second term on the right hand side are positive.