Discriminatory versus Uniform-Price Electricity Auctions with Supply Function Equilibrium

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Abstract. A goal of this paper is to compare results for discriminatory auctions to results for uniform-price auctions when suppliers have capacity constraints. We have a pretty good understanding of what equilibrium results look like for the uniform-price auctions. But an unresolved problem is what happens when a discriminative auction is run and suppliers have capacity constraints. We formulate a supply function equilibrium (SFE) model in continuous offer schedules with inelastic, time varying demand and with single step marginal cost function to compare two auction institutions in the presence of capacity constraints. We show that payments made to the suppliers in the unique equilibrium of the discriminatory auction can be less than the payments in the uniform-price auction, depending on which uniform-price auction equilibrium is selected. For the high demand and/or low excess capacity cases we also characterize mixed strategy supply function equilibrium under the discriminatory auction.

Keywords: Supply function equilibrium, continuous offer schedules, electricity markets, uniform-price auction, discriminatory auction.

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1. Introduction

The discriminatory or pay-as-bid auction institution was adopted by the British Regulatory Authority in the England and Wales wholesale electricity market in March 2001. In some US electric power markets (e.g., California power market) similar discussions are being held for changing the auction format from uniform to discriminatory pricing with the supposition that the discriminatory auction might achieve more efficient outcomes. In these environments, which auction institution yields higher profits for sellers, or lower average prices for buyers still remain open questions. Efficiency results and welfare rankings of these auctions in the electricity markets are being examined by various researchers such as Fabra, von der Fehr and Harbord (Ref. 1), Federico and Rahman (Ref. 2), Rassenti et al. (Ref. 3), Fabra (Ref. 4), Holmberg (Ref. 5), and Son, Baldick, Lee and Siddiqi (Ref. 6) under various assumptions.

A goal of this paper is to compare results for discriminatory auctions to results for uniform-price auctions when suppliers have capacity constraints. Take the case of constant marginal cost up to capacity, for example. We have a pretty good understanding of what equilibrium results look like for the uniform-price auctions. But an unresolved problem is what happens when a discriminative auction is run and suppliers have capacity constraints.

Fabra, von der Fehr and Harbord (Ref. 1) article is particularly important and is closely related to the research reported here. Most of the analysis of Fabra et al. is based on the assumption that bids are “short-lived” and are discrete step supply offers. However, in some markets suppliers can submit what amount to continuous, piece-wise linear, positively-sloped supply functions with quantity choices that determine the “elbow points” [see e.g., Wolfram’s (Ref. 7) description of the England and Wales auction in the early 1990’s]. Strategies of this type are also described by Hortacsu and Puller (Ref. 8) for the Texas electricity balancing market. Fabra et al.’s analysis and conclusions about the relative utility of discriminatory versus uniform-price electricity auctions are relevant and valuable in markets characterized by "short-lived" bids and rules requiring discrete supply functions. However, continuous supply function equilibrium (SFE) outcomes are quite different in markets characterized by "long-lived" bids. Genc and Reynolds (Ref. 9) developed a continuous SFE model that proves the existence of multiple pure strategy equilibria under the uniform-price auction institution in markets of this type. This stands in marked contrast to Fabra et al. step function approach which, for example, finds no pure strategy equilibrium in a parameter region of considerable empirical interest.

The model presented in this paper differs from the current literature in many ways. To see the differences, I explain closely related papers in detail. Federico and Rahman (Ref. 2) compare the two auction formats for two extreme cases: perfect competition and monopoly market structures. Under complete information over costs and fixed-perfectly-inelastic demand, they find that these two auctions result in the same prices and payoffs. However these results change if demand is inelastic and uncertain. Rassenti et al. (Ref. 3) experimentally analyze market outcomes of the discriminatory and uniform auctions. Each seller has multiple technologies with fixed capacities and submits step function offer schedule to the market. They find that changing auction format from uniform to discriminatory leads to significant electricity price increases in the off-peak and shoulder periods. They also find that the price variability from trading period to trading period is lower under the discriminatory auction than under the uniform-price auction when there is a greater excess capacity during the peak period. Wolfram (Ref. 10) conjectures in favor of the uniform-price auction in the England Wales Electricity Pool. Kahn et al. (Ref. 11) reject the idea of switching to the discriminatory auction, and claim that the discriminatory auction may cause inefficiencies, because generators will no longer bid at their marginal costs, and the tacit collusion that exists within the uniform auction may persist in the discriminatory auction. Evans and Green (Ref. 12) report that after shifting the auction format to discriminatory pricing wholesale electricity prices have decreased in England and Wales. However, Bower (Ref.

13) and Newbery (Ref. 14) argue that this decrease is due to the asset divestitures, increase in imports and excess market capacity.

The other related research is the Holmberg (Ref. 5) paper, which is a valuable contribution. Although there are many differences between his paper and this paper, some of Holmberg results could be viewed as complementary to my results, since they deal with a situation “capacity shortage” with positive probability. Holmberg compares the auction formats using completely inelastic demand. He assumes continuous marginal costs and derives SFE with the condition that demand exceeds total available industry capacity with positive probability. He claims that pure strategy equilibrium may not exist in the discriminatory auction, if demand has an increasing hazard rate and the marginal costs are constant, and concludes that average prices are weakly lower in the discriminatory auction. However, he does not deal with characterization of mixed strategy equilibrium in the discriminatory auction. Anwar (Ref. 15) also compares the discriminatory and uniform-price auctions. He studies equilibria in multi-unit common value auction model that sometimes provides a positive residual market demand to suppliers. He considers discrete step supply offers, and assumes uncertain demand. Each firm has the same constant marginal cost up to capacity. He shows that when demand is high and firms face some residual demand, the uniform auction leads to higher prices than the discriminatory auction. Fabra, von der Fehr and Harbord (Ref. 1) analyze a game-theoretic model in which firms with asymmetric capacities and costs submit discrete unit offer schedules to the auctioneer. Most of their analysis assumes a completely inelastic demand with a fixed market reserve price, constant marginal cost of production, and production capacity constraints. For the uncertain demand and perfectly symmetric case, they find that expected payments to suppliers are the same for both auctions. They also find that for low demand realizations, equilibrium is both unique and identical; the equilibrium is bidding at the marginal cost of the inefficient supplier for the two auction formats. For the asymmetric duopoly case, in the discriminatory auction they find that there is no pure strategy equilibrium but only in mixed strategies. Son, Baldick, Lee and Siddiqi (Ref. 6) compare performance of two strategic players, one is with large capacity the other is with small capacity, under both auction formats in a short term electricity market game. Players bid energy blocks and face inelastic or elastic demand in the auction. They show that contrary to the 'revenue equivalence theorem' players' expected total revenues are higher under uniform pricing than under the pay-as- bid pricing. They simulate the model results and compute the mixed strategy equilibrium under the discriminatory auction by applying the Lemke and Hawson algorithm, and discuss the likely effects of transmission constraints and multiple players on their findings.

In the paper I extend the SFE approach presented by Genc and Reynolds (Ref. 9) to compare discriminatory versus uniform-price electricity auctions. I assume that demand is time dependent (equivalently, stochastic) and is perfectly inelastic up to a price cap. I also assume that total industry capacity is greater than or equal to the peak demand. I compare the two auction formats when each firm's marginal cost of production is constant up to its production capacity. I consider symmetric (and asymmetric, whenever possible) Nash equilibrium in continuous supply function strategies in oligopoly. In this basic model, I find that in the discriminatory auction, the optimal equilibrium supply function is unique and suppliers bid competitively. However, in the uniform-price auction there is a continuum of equilibria as in other SFE models (such as Baldick and Hogan (Ref. 16), Day, Hobbs and Pang (Ref. 17), Anderson and Philpott (Ref. 18), Klemperer and Meyer (Ref. 19)) in which equilibrium prices range from marginal cost to the price cap. Therefore, each player's profit in the uniform auction is always weakly greater than the profit in the discriminatory auction at any time during the trading period. I also find that in the single-step marginal cost case, the functional form of the demand is irrelevant to the equilibrium strategies in both auction institutions. These results are consistent with the work of Back and Zender (Ref. 20), Wang and Zender (Ref. 21).

In addition this paper makes the following contributions. First, our findings are applicable to wholesale electricity auctions that are conducted under both "long-lived" and "short-lived" auction rules (see Section 2). Second, we consider the effects of capacity constraints and pivotal suppliers. We demonstrate that there is no pure strategy equilibrium in the discriminatory auction; however equilibrium strategies in the uniform-price auction are in pure strategies and multiple. By the pivotal suppliers analysis Genc and Reynolds (Ref. 9) show that some of the proposed SFE in the literature are not equilibrium under the uniform-price auction. In the current paper, I show that this result holds even under the discriminatory auction (for example, Wang and Zender type of equilibria can be ruled out). Third, in the asymmetric capacity model, for the high demand and/or low excess capacity cases we also characterize the mixed strategy Nash equilibrium in supply functions under the discriminatory auction. Fourth, given that bidding rules as well as auction formats are market design issues, we discuss that as a market policy the auctioneer should promote continuous supply function bidding. This policy would be useful for both the auctioneer and the suppliers especially for equilibrium predictions, since the step function bidding predicts mixed strategy equilibrium which is hard to compute and undesirable from the operational point of view (see Son et al. (Ref. 6)).

2. A Symmetric Supply Function Equilibrium Model

Let $n \geq 2$ be the number of suppliers and $N(t) = 1 - t$ be the time dependent and perfectly inelastic market demand (load duration curve) up to some exogenous choke price $\overline{p}$, where $t \in [0,1]$ is time in a trading period[1]. The deterministic variation in demand over time is mathematically equivalent to Klemperer and Meyer's (Ref. 19) model of uncertain demand with bounded variation. For this load function the corresponding stochastic demand would be $G(Q) = 1 - N^{-1}(Q)$ in which load quantity Q has support $[0,1]$. Let $K_i > 0$ be capacity for firm i,

[1] Demand changes over periods in a predictable way are also used by several authors including Anderson and Xu (Ref. 22), and Newbery (Ref. 23).

and assume that $K = \sum_i K_i \geq N(0)$, that is the total industry capacity can meet the peak load. Also let $z(s_i(p(t)))$ be piece-wise continuous marginal cost function, where $s_i(p)$ is firm *i*'s supply function. Each supplier *i* is required to submit a supply function prior to the realization of the load to maximize his profit; given that rivals' supply functions are fixed. Market clears at the lowest price that equates aggregate supply function to demand. Each firm is paid at that price for all capacity offers in the uniform-price auction. Each firm is paid at its bid price for the quantities supplied in the discriminatory auction. In case of excess supply at the clearing price, we assume a pro-rata on the margin rule for allocating excess supply (see Genc and Reynolds (Ref. 9)). If there is excess demand we assume there is no trade in that period.

Let $p(1)$, $p(0)$ be prices at times 1 and 0, respectively. Let $T(p) = 1 - \sum_i s_i(p)$ be the time at which price p would clear the market, and β be the auction format parameter. Specifically let $\beta = 1$ refer to the uniform auction, and $\beta = 0$ refer to the discriminatory auction. I assume players are symmetric, and I characterize pure strategy symmetric SFE. (Asymmetric equilibrium in mixed strategies with asymmetric players is considered in Section 3.) I assume that each supplier has a constant marginal cost of production up to capacity, that is $z(s(p)) = c$, where $0 \leq c < \overline{p}$. Note that our model could be applied to two different types of auction rules: long-lived and short-lived auctions that have been conducted in wholesale electricity markets.

Auction with long-lived bids: Each supplier must submit a single supply function that is binding for an entire day. Baldick and Hogan (Ref. 16, in their section 4.2) refer to this as auction rules requiring consistency of bids over the time horizon. Suppliers have a common understanding of what the load will be for each period (e.g., ¼ hour) of the day, but they cannot submit different supply functions for different (¼ hour) periods. The auctioneer determines a clearing price for each (¼ hour) period, and each supplier is obligated to produce up to their supply quantity at the clearing price for each ¼ hour) period. The load function $N(t)$ can be interpreted as a continuous approximation to the 96 load quantities for ¼ hour periods, ordered from highest to lowest.

Auction with short-lived bids: Suppliers are permitted to submit a supply function for each (¼ hour) period. The stochastic load interpretation of our model would be applicable if suppliers faced some uncertainty about the load quantity for these (¼ hour) periods at the time they submit their supply functions. There would be a different load ratio (i.e., ratio of minimum possible load to maximum possible load) for each (¼ hour) period.

Profit maximization problem of a firm in both auction formats can be formulated by using several methods, below we use a specific one. The other methods, such as the one in Wang and Zender (Ref. 21), lead to the same SFE conditions. Analogous to the method described in Rudkevich (Ref. 24), assume that for each time period t, and for each increment of capacity $ds(p)$ offered to the market at the prices range from p to $p+dp$, a firm is paid the bid price p and a price that is proportion of the market price and the bid price. Then the infinitesimal revenue flow will be, $dr(t,p)=[p+\beta(P(t)-p)]ds(p)$. Hence, the time t revenue for a firm is, $r(t)=s(p(1))[p(1)+\beta(P(t)-p(1))]+\int_{P(1)}^{P(t)}[x+\beta(P(t)-x)]ds(x)$. Subtracting time t cost and integrating time t profit over total trading period (i.e., [0,1]) will result in total profit. After some algebraic manipulations (by using several integration by parts) one can obtain the total profit for a firm during the trading period: $\pi(.)=-\int_{p(1)}^{p(0)}(p-z(.))s(p)T'(p)dp-(1-\beta)\int_{p(1)}^{p(0)}T(p)s(p)dp$.

Denote rivals' total supply $q(p)$, then the firm's optimization problem is

$$\begin{aligned}
&\min(-\pi(.)) \\
&s.t. \quad ds/dp=u(p)\ , \\
&\qquad q(p)+s(p)=N(t)\ , \\
&\qquad T(p(1))=1\ ,\ T(p(0))=0,\ u(p)\geq 0.
\end{aligned}$$

where the supplier can only choose a price at some time t indirectly, via their choice of supply function.

Lagrangian:

$$L(.)=(p-z)s(p)T'(p)+(1-\beta)T(p)s(p)+\psi(p)[s'(p)-u(p)]+\lambda(p)[q(p)+s(p)-N(t)]$$

Terminal function and necessary conditions are as follows.

Terminal function: $l(.)=-s(p(1))\mu+s(p(0))\eta$.

Lagrange-Euler equations: $\frac{d}{dp}\frac{\partial L}{\partial s'}=\frac{\partial L}{\partial s}$, $\frac{d}{dp}\frac{\partial L}{\partial T'}=\frac{\partial L}{\partial T}$.

Pontryagin optimality principle: $L=\min_{u\geq 0}L(.)$.

Transversality conditions: $\left.\frac{\partial L}{\partial s'}\right|_{p=p(1)}=\frac{\partial l}{\partial s(p(1))}$, $\left.\frac{\partial L}{\partial s'}\right|_{p=p(0)}=-\frac{\partial l}{\partial s(p(0))}$.

Complementarity conditions: $s(p(1))\mu=0$, $\mu\geq 0$, $s(p(0))\eta=0$, $\eta\geq 0$.

After solving Lagrange-Euler equations and Pontryagin principle, and rearranging terms we obtain the equation

$$s(p)+(p-z)s'(p)=(1-\beta)s(p)+N'(t)[(p-z)T'(p)+(1-\beta)T(p)] \qquad (1)$$

When $\beta=1$ and players are symmetric, for any $N(t)$, we obtain $s'(p)=\frac{s(p)}{(n-1)(p-z)}$, (since $T'(p)N'(t)=s'(p)n$). When $\beta=0$ and players are symmetric and $N(t)=1-t$, then (1) results in $s'(p)=\frac{1-s(p)n}{(n-1)(p-z)}$. Therefore, we obtain the following proposition.

Proposition 2.1: Symmetric equilibrium supply functions must satisfy the following conditions:

For the uniform-price auction $p'_{uni}(Q)=\frac{(n-1)(p(Q)-z(Q/n))}{Q}$ (2), for the discriminatory auction $p'_{disc}(Q) = \dfrac{(n-1)(p(Q) - z(Q/n))}{n(1-Q)}$ (3),

where Q is the industry supply.

Proposition 2.2: For symmetric and constant marginal cost case (and non-binding capacity constraints), SFE is unique and the equilibrium market price is equal to the marginal cost of production in the discriminatory auction and SFE is multiple and the range of equilibrium prices is in between marginal cost and price cap in the uniform-price auction.

Proof: In the case of $z(Q/n) = c$, that is the marginal cost of production is constant, where $Q \in (0,1)$, the solution of (2) satisfies, with the initial condition $p_{uni}(N(0)) \equiv p_{uni}(0)$,

$$s(p_{uni}) = \frac{1}{n}\left[\frac{p_{uni} - c}{p_{uni}(0) - c}\right]^{1/(n-1)} \qquad (4).$$

This is the commonly studied uniform-price auction SFE in the literature. In (4) a firm's equilibrium supply function is indexed by the initial price, $p_{uni}(0) \in (c, \overline{p}]$, and (4) does not violate the capacity constraints. (4) can also be written in as inverse industry equilibrium supply function,

$$p_{uni}(Q) = c + (p_{uni}(0) - c)Q^{n-1} \qquad (5).$$

For the discriminatory auction the solution of (3) yields, $p_{disc}(Q) = (Q-1)^{-1+1/n}[a_0 + \frac{(n-1)}{n}\int_0^Q c(x-1)^{-1/n}dx]$, where a_0 is an integration constant. This equation can also be written as $p_{disc}(Q) = c + a_1(Q-1)^{-1+1/n}$, where a_1 is another constant. For $a_1 = 0$, $p_{disc}(Q) = c$. For $a_1 > 0$ as $Q \to 1$, $p_{disc}(Q) \to \infty$. For $a_1 < 0$ as $Q \to 1$, $p_{disc}(Q) \to -\infty$. Hence there is only one economically plausible solution, which is

$$p_{disc}(Q) = c \qquad (6).$$

One can also show that this is the only unique solution by using the initial condition $p_{disc}(N(1)=0)=c$ [2].

Note that if the initial price at time 1 was $c \mp \varepsilon$, where $\varepsilon \geq 0$ then $p_{disc}(Q)=c \mp \varepsilon(1-Q)^{-1+1/n}$. For a fixed ε as $Q \to 1$, $p_{disc}(Q) \to \mp \infty$, hence $\varepsilon = 0$ must hold. □

Multiplicity of the equilibria in the uniform auction stems from the fact that there are multiple functions that pass through the profit maximizing points. Bids that are always inframarginal are irrelevant to determine equilibrium payoffs.

Corollary 2.1: In the symmetric firms with constant marginal cost case, each player's profit in the uniform auction is weakly greater than the profit in the discriminatory auction at any time during the trading period.

This corollary is a direct result of the Proposition 2.2. Because in the uniform auction, the market price is always greater or equal to the marginal cost of production, hence price-cost markup is greater than or equal to zero, and players submit non-negative and non-decreasing supply functions. Whereas in the discriminatory auction, price-cost markup is always equal to zero, no matter what proper supply functions are submitted profit for a firm becomes zero. Intuitively, Bertrand type equilibrium prevails because marginal cost is constant, i.e., independent of the level of output.

It appears that equilibrium supply functions are independent of demand functional form for both auction formats in the single-step marginal cost case, as long as demand is a

[2] If, as in my formulation, total capacity is greater than or equal to max load, then a solution to the differential equation (3) must have $p(Q=1)=z(1/n)$; that is, the boundary price associated with the max load must equal the marginal cost (mc). When this price condition is satisfied, $p'(Q=1)$ is equal to zero/zero; but using l'Hopital's rule one can show that $p'(Q=1)=z'(1/n)/(n(2n-1))$. A boundary price in excess of marginal cost yields a situation in which $p'(Q=1)$ is equal to a positive number divided by zero, which is undefined. This discussion explains why Wang and Zender find that when marginal cost is constant, the only equilibrium is to supply all units at the constant marginal cost.

differentiable function of time. As it can be seen from the equation (1) that in the uniform auction ($\beta = 1$), $s(p) = q'(p)(p-z)$ holds for any $N(t)$, since $T'(p)N_t' = s'(p) + q'(p)$, where $q(p)$ is the rivals' total supply function.

In the discriminatory auction ($\beta = 0$), the equation (1) takes the form of $-(p-z)q'(p) = N_t' \ T(p)$. For example, if demand was $N(t) = e^{-t}$, then by using the equation (1), $N_t' \ T(p) = ns(p)\ln(ns(p))$ holds for the symmetric case. Then the solution of this ordinary differential equation (ODE) yields $p_{disc}(Q) = c + a_2(\ln Q)^{-1+1/n}$. Using the similar argument that we used in the above proof, there is a unique solution of this expression, which only admits the constant $a_2 = 0$, hence $p_{disc}(Q) = c$. If demand was, say $N(t) = N(0)l^t$, where $1 \geq l > 0$, then the solution of (1) for $\beta = 0$ yields to, with symmetric players, $p_{disc}(Q) = c + a_3(\ln(Q/N(0))^{-1+1/n}$, which has again an economically meaningful solution only if $a_3 = 0$. Thus $\forall Q \in (N(1), N(0))$, $p_{disc}(Q) = c$. In the discriminatory auction modification of the demand functional form may change only the form of the second term in the inverse industry equilibrium supply function, $p_{disc}(Q)$. But the coefficient of the second term always gets to zero for an economically meaningful solution.

3. Extensions

3.1 Symmetric Firms with Capacity Constraints

In this section we consider the role of capacity constraints in equilibrium predictions. The following subsection regarding uniform-price auction is in the spirit of Genc and Reynolds (Ref. 9) and I will extend their approach to incorporate discriminatory auction. Without loss of generality let's make the following transformation of time period in the demand formulation so that we obtain one more variable in the analysis. That is, let time $t \leftrightarrow l\tau$ so that demand takes the form of $N(\tau)=1-l\tau$, where $l=1-N(1)/N(0)$ [3], which we call load factor and $l \in (0,1)$, $\tau \in [0,1]$. As defined earlier, the capacity index is $k \equiv K/N(0) \geq 1$, where $K=\sum_i K_i$ and K_i is the firm *i*'s total capacity. Also let marginal cost of production be constant up to the capacity.

Capacity constraints may eliminate some of the equilibria proposed in equations (5) and (6). Genc and Reynolds have studied this only for the uniform-price auction. In their analysis, some suppliers may tend to withhold some of their capacity from production, when demand is high and/or excess capacity is low in the market. These suppliers are called *pivotal suppliers*. Formally, a pivotal supplier j may withhold capacity during the trading period τ, if $\sum_{i\neq j} K_i < N(\tau)$. When this inequality holds, pivotal supplier j may sell the quantity $N(\tau)-\sum_{i\neq j} K_i$ at the price cap $\overline{p}$, and increase its profit. If $\sum_{i\neq j} K_i < N(1)$ holds, then each supplier is pivotal during the entire trading period. For the sake of simplicity, we will consider a simple deviation which implies that whenever rival firms do not have enough capacity to meet demand, a pivotal supplier will emerge and meet the residual demand at the price cap $\overline{p}$, and he will not supply any quantity below that price.

There are three exogenous parameters (k,l,n) that determine parameter regions. If $l<1-(n-1)k/n$ then each firm is pivotal at all times τ in [0,1]. We refer to this as totally pivotal[4] (TP) region, formally $\mathrm{TP} \equiv \{(k,l,n) \mid n \geq 2,\ k \in [1,\ n/(n-1)),\ l \in (0, 1-(n-1)k/n)\}$. Second situation is that each firm is pivotal for some trading period, which we call partially pivotal (PP) case, formally $\mathrm{PP} \equiv \{(k,l,n) \mid n \geq 2,\ k \in [1,\ n/(n-1)),\ l \in [1-(n-1)k/n,\ 1)\}$. A third state is called never pivotal (NP) case, in which n-1 combination of firms have enough capacity

[3] This demand is qualitatively same as the demand $N(t)$=1-t. By transforming the demand to $N(\tau)=1-l\tau$, we extend the dimension of the parameters' space from 2 to 3 to more extensively research the effects of capacity constraints in equilibrium predictions. Also this transformation helps us better compare the equilibrium predictions of this paper with that of Fabra et al. (Ref. 1).

[4] Here parameter regions are in the same vein of the regions depicted in Figure Two of Genc and Reynolds. The difference is that the load ratio curve is downward sloping of its argument k in the above definition.

to meet the peak load $N(0)$. Formally, $\text{NP} \equiv \{(k,l,n) \mid n \geq 2,\ \ k > n/(n-1),\ \ l \in (0,1)\}$, in which the pivotal supplier has no gain to deviate from the candidate optimal supply function.

3.1.1 Role of Capacity Constraints in the Uniform-price Auction

Given the functional forms we have assumed for demand and cost, we calculate, for each possible initial price, the SFE equilibrium prices and profit per firm

$$p_{uni}(\tau) = c + (p_{uni}(0) - c)(1 - l\tau)^{n-1},$$

$$\Pi_{uni}^{SFE} = \int_{\tau=0}^{1} (p(\tau) - c)(N(\tau)/n)d\tau = \frac{(p_{uni}(0) - c)(1-(1-l)^{n+1})}{n(n+1)l} \qquad (7),$$

where Π_{uni}^{SFE} is the profit per firm associated with a candidate SFE.

The simple type of deviation involves a supply function with no units offered for prices below the maximum price, $\overline{p}$, and all units up to capacity offered at the price $\overline{p}$. We compare the profit associated with this simple type of deviation with the profit from candidate SFE.

Let the parameters be in the totally pivotal region; $(k,l,n) \in \text{TP}$. Then the residual demand at time *t* for a deviating firm at price $\overline{p}$ is, $[N(\tau) - (n-1)K/n] > 0$. Total profit for the deviating firm is,

$$\Pi_{uni}^{D} = \int_{0}^{1} [N(\tau) - (n-1)K/n](\overline{p} - c)\, d\tau = \frac{(\overline{p} - c)[n(2-l) - 2(n-1)k]}{2n} \qquad (8).$$

Deviation profit exceeds profit associated with the candidate SFE if $\Pi_{uni}^{SFE} < \Pi_{uni}^{D}$, or equivalently if, $p(0) - c < \phi(k,l,n)(\overline{p} - c)$, where

$$\phi(k,l,n) \equiv [n(n+1)(2-l)l - 2(n^2-1)kl]/[2(1-(1-l)^{n+1}] \qquad (9).$$

If the markup at the initial price for a candidate SFE is less than the fraction ϕ of the maximum markup, $\overline{p} - c$, then the candidate SFE is not an equilibrium. Equation (9) provides a sufficient condition for eliminating certain candidate supply function equilibria as equilibria.

Remark 3.1: The market power index (i.e., ϕ function) is decreasing in the capacity index *k*, decreasing in the load factor *l*, and decreasing in the number of suppliers *n* for $(k,l,n) \in \text{TP}$.

We omit the proof of this remark since it is analogous to the Proposition 2 in Genc and Reynolds (The difference is, in Genc and Reynolds the index is increasing in the load factor *l*). The economic interpretation of this proposition is straightforward.

Now let the parameters be in the partially pivotal region; $(k,l,n) \in \text{PP}$. Then similar to the above remark it can be easily shown that market power index for the PP region is decreasing in the capacity index *k*, decreasing in the load factor *l*, and decreasing in the number of suppliers n.

Some of the results we find above may be easily compared to those of Fabra et al. (Ref. 1) for the case of a uniform price auction with long-lived bids and two symmetric suppliers according to the above parameter regions. First, in the never pivotal (NP) region, each firm has enough capacity to serve the entire peak load. Fabra et al. find a unique Nash equilibrium in which each supplier bids at marginal cost. This Bertrand-like result is a much more competitive prediction than the continuous SFE approach yields. As defined in equation (4) supply function equilibria always involve market clearing prices above marginal cost, and include the least competitive SFE (with initial price equal to the market reserve price, $\overline{p}$). The Fabra et al. prediction for the NP region holds regardless of how many discrete units each supplier is permitted to submit offers for. Second, in the totally pivotal (TP) region, neither firm has enough capacity to serve the off peak load; at least some of the capacity of each firm is required to meet demand at all times during a trading period. Fabra et al. find multiple pure strategy equilibria, but all such equilibria yield a market clearing price equal to the market reserve price, $\overline{p}$. This is a more collusive prediction than that of the continuous SFE model, for which equilibrium prices are contained in an interval with upper bound equal to $\overline{p}$. Third, in the partially pivotal (PP) region, any single firm is pivotal for part, but not all, of the trading period. For step function bidding, Fabra et al. find that a pure strategy equilibrium does not exist for parameters in the PP region [see Lemma 3, Fabra et al., p. 35]; the equilibrium is in mixed strategies. Fabra et al. do not have characterization results for the mixed strategy equilibrium, except that the equilibrium price distribution approaches marginal cost pricing as parameters approach the NP region and that the equilibrium price distribution approaches pricing at the market reserve price with probability one as parameters approach the TP region. In the continuous SFE model there are multiple equilibria. The least competitive equilibrium yields an interval of prices with upper bound $\overline{p}$. The most competitive equilibrium has prices above marginal cost; how far above marginal cost depends on load factor and capacity index parameters. We state that predicted market clearing prices for the step function model may be either higher or lower than equilibrium SFE market clearing prices depending on parameter values, when parameters are in the PP region.

3.1.2 Role of Capacity Constraints in the Discriminatory Auction

In this section we show that the only pure strategy equilibrium in the discriminatory auction may be ruled out because of the capacity constraints and the role of pivotal suppliers. This suggests characterization of mixed strategy equilibrium in the discriminatory auction with capacity constrained firms.

Suppose that parameters (k, l, n) are in the NP region, in that case equation (3) does not violate capacity constraints and admits a unique solution. That is, inverse industry supply function is differentiable and the solution is $p_{disc}(Q) = c$. Since (n-1) rival suppliers have enough capacity to meet the highest level of load, there will be no incentive to deviate from marginal cost pricing. If a supplier offers a price above c then other firms already have incentives to undercut this supplier’s bid slightly so that they ensure to be called out. Also since no supplier faces positive residual demand, any marginal price (i.e., the price of the marginal unit that equates supply to demand) above the marginal cost is eliminated as pure strategy equilibrium.

When the parameters (*k*, *l*, *n*) are in the TP or the PP regions, then there is an incentive to deviate from marginal-cost-pricing. That is whenever capacity constraints bind, locally optimal offer schedules would not be globally optimal because profit function of deviating supplier becomes discontinuous and non-concave in price. We discuss this next.

Suppose that the parameters (*k*, *l*, *n*) are in the TP region, which means that the rival firms do not have enough capacity even to meet the off-peak load $N(1) = 1 - l$. Each firm's some portion of capacity is needed. Then a pivotal supplier at time τ faces the residual demand $N(\tau) - \sum_{i \neq j} K_i > 0$ at the price $\overline{p}$. Its deviation profit at time 0 is $\Pi_{disc}^{D}(.) = \int_{1-l}^{1} (\overline{p} - c) dQ = (\overline{p} - c) l$.

Obviously, $\Pi_{disc}^{D} > \Pi_{disc}^{SFE}$, where SFE profit is $\Pi_{disc}^{SFE} = 0$ since $p_{disc}(Q) = c$ for all $Q \leq K$. Hence marginal cost pricing cannot be equilibrium and the deviation leads to the marginal price $\overline{p}$. If parameters (*k*, *l*, *n*) are in the PP region, then a deviating firm's profit at the trading period zero is

$$\hat{\Pi}_{disc}^{D} = \int_{K(n-1)/n}^{1} (\overline{p} - c) dQ = [1 - K(n-1)/n](\overline{p} - c) > 0 = \Pi_{disc}^{SFE}.$$

Hence during the periods 0 through τ', where $N(\tau') = (n-1)K/n$, the marginal cost pricing suggested by $p_{disc}(Q) = c$ for all $Q \leq K$ is not part of the equilibrium.

The rivals' total capacity is not enough to meet the maximum load, with a positive probability a firm will face a residual demand in the PP region, with sure probability a firm will face a residual demand in the TP region. But this is true for each firm in the discriminatory auction; hence there will not be a pure strategy equilibrium. Thus, we have the following proposition.

Proposition 3.1: If parameters (k, l, n) are in the TP or the PP regions, in which the demand is higher than the rivals' total capacity for at least some trading period, then deviation incentives rule out the only equilibrium candidate in the discriminatory auction.

3.2 Mixed Strategy SFE with Asymmetric Suppliers in the Discriminatory Auction

Horizontal Supply Functions in TP Region:

In the above section we have seen that simple deviation rules out the only candidate pure strategy SFE when the parameters are in the TP region. It is also not optimal when all firms offer their generation units at the price cap since deviation is profitable. In this section we prove that it is equilibrium in the mixed strategies to bid a single price for the entire capacity of a supplier.

Assume n=2 asymmetric suppliers that face time-varying load $N(\tau)=1-l\tau$ such that $K_1+K_2 \geq N(0)$, $K_1 \leq N(1)$ and $K_2 \leq N(1)$. Also assume that firm 2 chooses a flat price p_2 (which is a continuous random variable) in $(c,\overline{p})$ for his entire capacity with the probability distribution $F(p_2)$. $F(p_2)$ is an atomless distribution function with support $[c,\overline{p}]$. Let p_1 be the price chosen by firm 1 for all of his capacity offered at time t in the trading period. Let k_1 and k_2 be capacity indices of firms 1 and 2, respectively. Note that $k_i = K_i$, $i=1,2$, since $N(0)=1$. These parameters naturally lead to the TP set.

Proposition 3.2: Assume that suppliers $i=1,2$ are allowed to submit step supply function bids. Let $k=k_1+k_2$. Any price offer $p_i \in (c,\overline{p})$ for the entire capacity of supplier i ($i=1,2$) with the probability distribution $F(p_i)=\dfrac{(\overline{p}-c)(k-1+l/2)-(\overline{p}-p_i)k_i}{(p_i-c)(k-1+l/2)}$ constitutes a mixed strategy Nash equilibrium in the TP region under the discriminatory auction. The auctioneer's total expected payment to the suppliers is $(\overline{p}-c)(2-l-k)+c(1-l/2)$ during the trading period.

Proof: The expected profit of supplier 1 during the entire trading period is,

$$\pi_1(p_1) = \int_0^1 \int_{p_1(t)}^{\overline{p}} [p_1(t) - c]k_1 dF(p)dt + \int_0^1 \int_c^{p_1(t)} [p_1(t) - c][N(t) - k_2]dF(p)dt$$
$$= [1 - F(p_1)][p_1 - c]k_1 + F(p_1)[p_1 - c][1 - k_2 - l/2]$$

in which we used the boundary conditions $F(\overline{p}) = 1$ and $F(c) = 0$. Above the first expression is the profit that stems from when the rival firm chooses a price above p_1. In that case firm 1 dumps his entire capacity to the market. The second expression of the profit is regarding the possibility that firm 2 chooses a price below p_1, and then firm 1 meets the residual demand, which is equal to the expected load ($1 - l/2$) minus the rival's supply, k_2.

In equilibrium $\pi_1'(p_1) = 0$ holds and it implies the probability distribution function

$$f(p_1) = \frac{(\overline{p} - c)(1 - k_2 - l/2)}{(p_1 - c)^2 (k - 1 + l/2)} \qquad (10),$$

and the cumulative distribution function

$$F(p_1) = \frac{(\overline{p} - c)(k - 1 + l/2) - (\overline{p} - p_1)k_1}{(p_1 - c)(k - 1 + l/2)} \qquad (11).$$

Substituting this probability into the profit function gives $\pi_1(p_1) = (\overline{p} - c)(1 - k_2 - l/2)$, which is independent of $p_1 \in (c, \overline{p})$ and supplier 1 can randomize the price, in this interval, with the probability in (11).

Since the game is symmetric we can perform a similar analysis for player 2 and obtain the probability functions,

$$f(p_2) = \frac{(\overline{p} - c)(1 - k_1 - l/2)}{(p_2 - c)^2 (k - 1 + l/2)}, \quad F(p_2) = \frac{(\overline{p} - c)(k - 1 + l/2) - (\overline{p} - p_2)k_2}{(p_2 - c)(k - 1 + l/2)}.$$

Player 2's profit will be $\pi_2(p_2) = (\overline{p} - c)(1 - k_1 - l/2)$, which is independent of $p_2 \in (c, \overline{p})$, and he can randomize any price in this interval.

Now we need to show that these mixed strategies form equilibrium. In the above we have shown that the strategies satisfy the first order necessary conditions. Next we will prove that if supplier i uses mixed strategies then supplier j's ($j \neq i$) best response cannot be increasing step supply function. The following proof is in the spirit of Anwar (Ref. 15). Assume that firm 2 chooses his mixed strategies in the price range $(c, \overline{p})$ according to the above probability function. Let $(b_j, q_j)_j$ be the sequence of bid prices and quantities for supplier 1, where $\sum_{j=1}^{m} q_j = k_1$, and $\overline{p} \geq b_1 > b_2 > ... > b_m > c$. Also let $b = (b_j)_j$.

Then the total expected profit for firm 1 during the trading period is,

$$
\begin{aligned}
\pi_1(b) = & \int_0^{t_1} \left\{ \int_{b_1}^{\overline{p}} \sum_{j=1}^{m} q_j (b_j - c) dF(p) + \int_c^{b_1} \left(\sum_{j=2}^{m} q_j (b_j - c) + \left[(N(t) - k_2) - (k_1 - q_1) \right] (b_1 - c) \right) dF(p) \right\} dt \\
& + \int_{t_1}^{t_2} \left\{ \int_{b_1}^{\overline{p}} \sum_{j=1}^{m} q_j (b_j - c) dF(p) + \int_{b_2}^{b_1} \sum_{j=2}^{m} q_j (b_j - c) dF(p) \right. \\
& \qquad \left. + \int_c^{b_2} \left(\sum_{j=3}^{m} q_j (b_j - c) + \left[(N(t) - k_2) - (k_1 - q_1 - q_2) \right] (b_2 - c) \right) dF(p) \right\} dt \\
& + ... \\
& + \int_{t_{m-1}}^{1} \left\{ \int_{b_1}^{\overline{p}} \sum_{j=1}^{m} q_j (b_j - c) dF(p) + \int_{b_2}^{b_1} \sum_{j=2}^{m} q_j (b_j - c) dF(p) + \sum_{r=3}^{m} \left[\int_{b_r}^{b_{r-1}} \left(\sum_{j=r}^{m} q_j (b_j - c) \right) dF(p) \right] \right. \\
& \qquad \left. + \int_c^{b_m} \left[(N(t) - k_2) - (k_1 - \sum_{j=1}^{m} q_j) \right] (b_m - c) dF(p) \right\} dt,
\end{aligned}
$$

where the first integration refers to the load interval for which the profit is calculated. The second integration along with the summation represents expected profit from the sales of each unit of quantity q_j at a price of b_j, given the price chosen by rival. If firm 2 chooses a price (with some probability) above the firm 1's maximum bid price, then firm 1 sells each unit of capacity at varying bid prices. Firm 2 also sells some quantity to the market since neither firm has enough capacity to meet even the off-peak load. The interpretation of other expressions is also similar.

Note that the above profit function is separable in terms of bid prices for the bid steps. That is, $\pi_1(b) = \sum_j \pi_1(b_j)$, where

$$\pi_1(b_1) = \int_0^{t_1} \left[q_1(b_1 - c) + F(b_1) \left(\sum_{j=2}^{m} q_j(b_j - c) - \sum_{j=1}^{m} q_j(b_j - c) + \left[(N(t) - k_2) - (k_1 - q_1) \right](b_1 - c) \right) \right] dt$$
$$+ \int_{t_1}^{t_2} \left[q_1(b_1 - c) + F(b_1) \left(\sum_{j=2}^{m} q_j(b_j - c) - \sum_{j=1}^{m} q_j(b_j - c) \right) \right] dt$$
$$+ \ldots$$
$$+ \int_{t_{m-1}}^{1} \left[q_1(b_1 - c) + F(b_1) \left(\sum_{j=2}^{m} q_j(b_j - c) - \sum_{j=1}^{m} q_j(b_j - c) \right) \right] dt.$$

This profit expression simplifies to

$$\pi_1(b_1) = [1 - F(b_1)] q_1(b_1 - c) + \int_0^{t_1} F(b_1)(b_1 - c)\left[(N(t) - k_2) - (k_1 - q_1) \right] dt,$$

where $F(b_1)$ is as in (11) with $b_1 = p_1$. We take the first derivative of the profit function and obtain,

$$\frac{d\pi_1(b_1)}{b_1} = \frac{k_1 \left\{ \int_0^{t_1} (N(t) - k) dt - \int_{t_1}^{1} q_1 dt \right\} + q_1(k - 1 + l/2)}{k - 1 + l/2} = \frac{q_1(k_2 + k_1 t_1 - 1 + l/2) - k_1(kt_1 - t_1 + lt_1^2/2)}{k - 1 + l/2}.$$

Let $Y \equiv k_2 + k_1 t_1 - 1 + l/2$ and $Z \equiv kt_1 - t_1 + lt_1^2/2$.

First note that if $t_1 = 0$ then $Z = 0$, and if $t_1 > 0$ then $Z > 0$. Next,

$$Z - Y = (1 - k_2)(1 - t_1) - \frac{l}{2}(1 - t_1^2) = (1 - t_1)[1 - k_2 - \frac{l}{2}(1 + t_1)] > (1 - t_1)[1 - k_2 - l] \geq 0,$$

with strict inequality for $t_1 < 1$. The inequalities involving Z and Y, combined with the conditions $q_1 < k_1$ and $k - 1 + l/2 > 0$, imply that $\frac{d\pi_1(b_1)}{db_1} < 0$.

This derivative implies that firm 1 should reduce its bid price from b_1 to b_2. Then firm 1 will have m-1 bid prices. Repeating the above process (m-2) times results in a single price offer, which is in the interval $(c, \overline{p})$, for all units of his capacity. Thus, optimal response to the rival's single-price-offer is to submit one price for all units of generation. If we perform the similar analysis for firm 2, we obtain that firm 2 also submits a single price offer for his entire capacity.

Given that $0 < F(p_1), F(p_2) < 1$, $\pi_1(p_1)$ and $\pi_2(p_2)$ are twice differentiable and concave in prices and when supplier *i* uses mixed strategies then supplier *j*'s ($j \neq i$) best response cannot be increasing step supply function, we conclude that $F(p_i) = \dfrac{(\overline{p}-c)(k-1+l/2)-(\overline{p}-p_i)k_i}{(p_i-c)(k-1+l/2)}$ constitutes a mixed strategy Nash equilibrium in the TP region under the discriminatory auction for $i = 1, 2$.

The auctioneer's total expected payment to the players will be, $\pi_1(p_1) + \pi_2(p_2) + c\int_0^1 N(t)dt = (\overline{p}-c)(2-l-k) + c(1-l/2)$, where the first term is firm 1's profit, the second term is firm 2's profit, and the final term is the total cost of production in the entire trading period. □

<u>*Smooth Supply Function Case*</u>:

Now consider the case in which suppliers are allowed to bid in continuous supply function strategies (instead of step supply functions), and a rival supplier uses a mixed strategy. Then we claim that one cannot construct a smooth increasing offer function that yields higher expected profit than the expected profit from any horizontal offer. The intuition behind that is that by choosing a sufficiently large number of steps, one could come arbitrarily close to the profits obtainable from a smooth supply function by using a multi-step offer function. But in the proof of Proposition 3.2 we show that multi-step offer function yields lower profit than a single step offer function. Thus, the optimal response to the rival's mixed strategy cannot be a smooth supply function.

Horizontal Supply Functions in PP Region:

Now consider the case such that for $i = 1,2$, $1 - l\tau = N(\tau) \geq k_i$, if $\tau \in (0,t]$ and $N(\tau) < k_i$, if $\tau \in (t,1]$. Also assume that $N(0) \leq k_1 + k_2$. This case represents the PP set.

Proposition 3.3: Assume that suppliers are allowed to submit step supply function bids. Let k_i be the capacity index of supplier *i* ($i = 1,2$) and let $k = k_1 + k_2$. There exists a mixed strategy Nash equilibrium in the discriminatory auction in the PP region through the price $p_i \in (c, \overline{p})$ offered for the entire capacity of supplier *i* with the probability function,

$$F(p_i) = \frac{(\overline{p} - c)(1 - 2t + lt^2 - l/2 + tk) - (\overline{p} - p_i)(1 - t + lt^2/2 - l/2 + tk_i)}{(p_i - c)(1 - 2t + lt^2 - l/2 + tk)}.$$

Proof: Suppose that (supplier 1 believes that) supplier 2 chooses a price $p_2 \in (c, \overline{p})$ with probability $F(p_2)$ and supplier 1 simultaneously responds to rival's strategy with a price p_1 for his entire capacity. Then the expected profit of supplier 1 during the trading period is,

$$\begin{aligned}
\pi_1(p_1) &= \int_0^t \left[\int_{p_1(t)}^{\overline{p}} [p_1(t) - c]k_1 dF(p) + \int_c^{p_1(t)} [p_1(t) - c][N(t) - k_2] dF(p) \right] dt \\
&\quad + \int_t^1 \left[\int_{p_1(t)}^{\overline{p}} [p_1(t) - c]N(t) dF(p) + \int_c^{p_1(t)} 0 dF(p) \right] dt \\
&= [1 - F(p_1)][p_1 - c]k_1 t + F(p_1)[p_1 - c][t - lt^2/2 - tk_2] \\
&\quad + [1 - F(p_1)][p_1 - c][1 - l/2 - t + lt^2/2],
\end{aligned}$$

where *t* solves for $1 - lt = k_2$.

In equilibrium $\pi_1'(p_1) = 0$ holds and it implies the probability and the cumulative distribution functions

$$f(p_1) = \frac{(\overline{p} - c)(A_1 - B)}{(p_1 - c)^2 (B)}, \quad F(p_1) = \frac{(\overline{p} - c)(B) - (\overline{p} - p_1)(A_1)}{(p_1 - c)(B)}, \text{ respectively,}$$

where $A_1 = 1 - t + lt^2/2 - l/2 + tk_1$, and $B = 1 - 2t + lt^2 - l/2 + tk$.

Since the game is symmetric we can perform similar analysis for player 2 and obtain the probability functions,

$$f(p_2)=\frac{(\overline{p}-c)(A_2-B)}{(p_2-c)^2(B)} \quad \text{and} \quad F(p_2)=\frac{(\overline{p}-c)(B)-(\overline{p}-p_2)(A_2)}{(p_2-c)(B)},$$

where $A_2=1-t+lt^2/2-l/2+tk_2$, and B is as above.

Next we will show that if supplier 2 uses mixed strategy then supplier 1's best response cannot be increasing step supply function in prices. Assume that firm 2 chooses his strategies in the price range $(c,\overline{p})$ according to the above probability function.

We form the horizontal supply strategies as follows. For simplicity assume that firm 1 chooses two price-quantity pairs, $(b_j,q_j)_j$, $j=1,2$, where $\sum_{j=1}^{m=2} q_j=k_1$, and $\overline{p}\geq b_1>b_2>c$. Also let $b=(b_j)_j$. One can easily extend this analysis to *m*>2 price-quantity pairs, as we did above. The total expected profit for firm 1 during the trading period is,

$$\begin{aligned}
\pi_1(b)=\int_0^{t_1}\Bigg\{&\int_{b_1}^{\overline{p}}\left[q_1(b_1-c)+(k_1-q_1)(b_2-c)\right]dF(p)\\
&+\int_c^{b_1}\Big[(k_1-q_1)(b_2-c)+\left[(N(t)-k_2)-(k_1-q_1)\right](b_1-c)\Big]dF(p)\Bigg\}dt\\
+\int_{t_1}^{t_2}\Bigg\{&\int_{b_1}^{\overline{p}}\left[q_1(b_1-c)+(k_1-q_1)(b_2-c)\right]dF(p)+\int_{b_2}^{b_1}(k_1-q_1)(b_2-c)dF(p)\\
&+\int_c^{b_2}(N(t)-k_2)(b_2-c)dF(p)\Bigg\}dt\\
+\int_{t_2}^{t_3}\Bigg\{&\int_{b_1}^{\overline{p}}[[N(t)-(k_1-q_1)](b_1-c)+(k_1-q_1)(b_2-c)]dF(p)+\int_{b_2}^{b_1}(k_1-q_1)(b_2-c)dF(p)\Bigg\}dt\\
+\int_{t_3}^{1}\Bigg\{&\int_{b_1}^{\overline{p}}N(t)(b_2-c)dF(p)+\int_{b_2}^{b_1}N(t)(b_2-c)dF(p)\Bigg\}dt.
\end{aligned}$$

The above profit function is separable in terms of bid prices for the steps. That is, $\pi_1(b) = \sum_j \pi_1(b_j)$. After some simplifications we write the profit function in terms of b_1 as follows:

$$\pi_1(b_1) = F(b_1)\int_0^{t_1}[N(t) - k_2 - (k_1 - q_1)](b_1 - c)dt$$
$$+[1 - F(b_1)]\int_{t_2}^{t_3}[N(t) - k_1](b_1 - c)dt + [1 - F(b_1)]t_3 q_1(b_1 - c),$$

where $F(b_1)$ is as in above with $b_1 = p_1$. We take the first derivative of the profit function and obtain,

$$\frac{d\pi_1(b_1)}{db_1} = f(b_1)\int_0^{t_1}[N(t) - k_2 - (k_1 - q_1)](b_1 - c)dt$$
$$+F(b_1)\int_0^{t_1}[N(t) - k_2 - (k_1 - q_1)]dt + t_2 q_1[-f(b_1)(b_1 - c) + 1 - F(b_1)]$$
$$+[-f(b_1)(b_1 - c) + 1 - F(b_1)]\int_{t_2}^{t_3}[N(t) - (k_1 - q_1)](b_1 - c)dt.$$

We find that $[-f(b_1)(b_1 - c) + 1 - F(b_1)] = (B - A_1)/B < 0$, where A_1, B are as defined above and $B - A_1 = t[k_2 - 1 + lt/2] \leq -t^2 l/2 < 0$, and $B > 0$. Also $N(t_1) = k - q_1$ and $N(t_2) = k_2$ hold. Hence, in the above the first two terms are positive and the rest is negative. If $k_1 - q_1$ is large, then the first two terms approach to zero since $t_1 \to 0$. Therefore $\frac{d\pi_1(b_1)}{db_1} < 0$. Subsequently we conclude that offering the entire capacity at a single price is more profitable than using a step function with multiple bid prices for the quantity steps. □

4. Conclusions

We examine generators' bidding behavior in the uniform and discriminatory price auctions under various assumptions on equilibrium bidding types, and capacity constraints. We discuss the difficulties, from operational and computational points of view, of the mixed strategy equilibrium in the market designs allowing discrete offers, even though both types of bidding rules have different predictions. Some of our results in the basic model are consistent with the current literature. We show that the discriminatory auction SFE is unique, but the equilibrium is multiple in the uniform auction. When capacity constraints bind and pivotal suppliers face positive residual demand we obtain that there is no pure strategy SFE in the discriminatory auction. In the mixed strategy supply function analysis we examine the nature of a best response to a rival strategy that is a mixed strategy over horizontal supply functions. We show that offering all of the capacity at a single price is more profitable than using a step function with multiple bid prices for the steps. We conclude that it is not clear whether the discriminatory auction format is tractable for the suppliers and/or the auctioneer or whether it entails low prices for the consumers.

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