UNIVERSITY of GUELPH
MATHEMATICS & STATISTICS

MATH*4240F19 - Course Outline
Advanced Topics in Modeling and Optimization
University of Guelph, Guelph Campus

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Course Calendar Description

This course is a study of advanced topics in the areas of optimization and modeling. Topics may include continuous and discrete models together with techniques for their analysis and design, and optimization topics such as game theory, networks, nonlinear problems, Markov chains, queuing theory, agent-based models, computational intelligence based techniques and computational optimization techniques.

- Prerequisite(s): 0.50 credit in a mathematics course at the 3000 level.

Super-set of Likely Topics

Convexity, Birkhoff polytope, Extended Formulations & Projections, complexity theory, simplex method & applications, duality, Karmarkar algorithm, integer programming, probability, decision theory, dynamic programming, Markov chains, queuing theory, Farkas lemma, Kuhn-Tucker conditions, non-linear programming, optimization (gradient method).

Required Study Materials

- **Operations Research**, Second Edition, Shaum's Outline Series, Richard Bronson and Govindasami Naadimuthu, McGraw-Hill, © 1997. ISBN: 0-07–008020-8 (paperback book).
- **Graph Theory**, Shaum's Outline Series, V.K. Balakrishnan, McGraw-Hill, © 1997. ISBN: 0-07–005489-4 (paperback book)
- Your own laptop / notebook AND purchase of Maple 2019 Student Edition (software) by MapleSoft available for download at https://guelph.onthehub.com/WebStore/Welcome.aspx. Alternatively, goto https://webstore.maplesoft.com and use the promotional code AP21835-B0FA6.

Learning Outcomes

- Display knowledge of: the definition of a convex set, the definition of a convex polytope, the definition of a convex function, Caratheodory's theorem and proof.
- Ability to implement Fourier-Motzin technqiues, and explain the concept of 1) a flat versus full-volume polytope and, 2) an orthogonal projection of a polytope and its image polytope
- Ability to contrast Simplex method and Karmarkar algorithm, model and solve problems as LPs, formulate the primal and dual formulation as a single LP.
- Display a fundamental understanding of BM, GI and HC and related graph theory definitions used to define these problems.
- Display a fundamental understanding of complexity theory and be able to classify BM, GI and HC and TSP.
- Display a fundamental understanding in four of the associated "List of Topics" – see Assessment.

Lecture Times

- Tuesday and Thursday 16:00 – 17:20 in MCKN 308.
- FOUR LECTURES NEED TO BE RESCHEDULED – see last page for details

The first day of classes is Thursday September 5. There is no class on Tuesday October 15. The last day of classes is Thursday November 28.

Office Hours

- TTh 17:30 – 19:30 in MACN 510

Assessment

- There are five assignments each worth 10%.
- There is one 72 hour take-home final examination worth 50%.

The first three assignments are problem-solving assignments – based upon materials originating from

lecture. You are encouraged to work together. Please submit independent write-ups. The fourth assignment is a problem solving / theorem proving based assignment. TENTATIVE PLAN: I'll provide 10 independent Theoretical Questions (below) and assign these questions to students. You can work together if you like. But the goal is for each of you to do as much as you can, and submit independent write-ups. Guidelines forthcoming. The fifth assignment is group-work (4 to a group) and class presentation. That is, you'll be assigned to groups, including your individual responsibilities within the group. I'll provide a List of Topics (below) that your group can choose from - to be presented in class. Details forthcoming.

Theoretical Questions (more to come)

1. Given simple graphs **G** and **H**, decide whether or not there exists a strict convex combination of permutation matrices $P = \alpha_1 P_1 + \alpha_2 P_2 + \ldots + \alpha_n P_n$ s.t. $P\mathbf{G}P^t = \boldsymbol{H}$. Prove it.
2. Given convex set K and number m, how can you arrange m supporting hyperplanes such that the intersection of the half spaces containing K is as close to K (in the Hausdorff metric) as possible. (borrowed problem from a colleague). Make a few ideas and show it.

List of Topics

Contrast the Birkhoff / TSP polytopes. Anything you want.
- interpretation as to how the TSP polytope captures complexity
- History of TSP polytope … ref. papers pls.

König–Egerváry Theorem.
- what is it & history & relevance?
- example / application

Farkas Lemma.
- what is it & history & relevance?
- example / application

Kuhn-Tucker conditions.
- what are they & history & relevance?
- example / application

coNP-complete.
- explain relative to NP-complete & relevance
- example / application
- look for models of coNP-complete problems

Bipartite Matching Algorithm.
- what is it & history & relevance?
- example / application

Method of Lagrange Multiplies.
- what is the problem and method
- example / application

Four Colour Theorem (4CT).
- what is it & history (rich) & relevance?
- example / application
- Is a computer based proof actually a mathematical proof? (open)

Satisfiability (SAT).
- what is it & history (also rich a.k.a. Stephen Cook) & relevance?
- example / application

Constraint satisfaction problem (CSP).
- what is it & history & relevance?
- example / application
- check out William Cook's work at Waterloo

Klee-Minty Polytope.

- what is it & history & relevance
- example / application

Describe the quantum computing paradigm & quantum complexity theory.

- example / application for any problem
- Does **P=NP** under the quantum computing paradigm? Check out Scott Aaronson, online.

Public Key Cryptography.

- what is it & history & relevance
- example and application
- check this out https://www.gp-digital.org/world-map-of-encryption/ together with 1) Is public key secure? and 2) anything you can find on government capabilities to snoop

Crypto Currency.

- what is it & history & relevance
- example / application (complexity to break?)

Matroid(s).

- what is (are) it (they) & history & relevance
- example / application

Reference Study Materials

- **Operations Research: An Introduction**, 10th edition by Hamdy A. Taha, Pearson, © 2017. ISBN: 0-13–444401-9 (hardcover book)
- **Linear programming and extensions**, George B. Dantzig, the Rand Corporation and University of California, Berkeley 1961.
- **Nonlinear programming : theory and algorithms**, Bazaraa, M. S. & Shetty, C.M., New York, Wiley, 1979.
- **An Introduction to Convex Polytopes**, A. Brondsted, Springer-Verlag, New York, 1983
- Cook, S. The complexity of theorem-proving procedures. STOC '71: Proceedings of the third annual ACM symposium on Theory of computing, May 1971
- **Godel, Escher and Bach: an Eternal Golden Braid** by Basic Books Inc. (more recently Douglas R. Hofstadter. ISBN 0-465-02656-7
- Karmarkar, N. A new polynomial-time algorithm for linear programming. Combinatorica 4 (1984), no. 4, 373-395
- Swart, E.R. How I implemented the Karmarkar algorithm in one evening, ACM SIGAPL,Volume 15 , Issue 3 (March 1985), 13-16
- Pickel, P. F. An update on the ellipsoid algorithm. Discrete geometry and convexity (New York, 1982), 364-380, Ann. New York Acad. Sci., 440, New York Acad. Sci., New York, 1985.
- Sipser, M. The History and Status of the P=NP Problem. 24th ANNUAL ACM STOC - 5/92/VICTORIA, B. C., CANADA, 1992 ACM
- Michael J. Todd. The many facets of linear programming. Math. Program., Ser. B 91 (2002) 417–436
- Balas, E. Projection lifting and extended formulation in integer and combinatorial optimization, Ann. Oper. Res.,140 (2005), 125-161
- Jack Edmonds, Paths, Trees, and Flowers, Canadian Journal of Mathematics. Volume 17, 449-467 (1965)
- K. Appel and W. Haken. Every planar map is four colorable. Bull. Amer. Math. Soc., Volume 82, Number 5 (1976), 711-712.
- E. R. Swart. The Philosophical Implications of the Four-Color Problem. The American Mathematical Monthly. Vol. 87, No. 9 (Nov., 1980), pp. 697-707
- Pulleyblank, W. Edmonds, Matching and the Birth of Polyhedral Combinatorics. https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=7&ved=2ahUKEwivpa_24KXjAhVFeawKHbINCxwQFjAGegQICBAC&url=https%3A%2F%2Fwww.math.uni-bielefeld.de%2Fdocumenta%2Fvol-ismp%2F34_pulleyblank-william.pdf&usg=AOvVaw2TZYHuFIbEhfjrIf8aI3O3

Related Online Talks and Conferences

- Symposium on 50 Years of Complexity Theory: A Celebration of the Work of Stephen Cook, Fields Institute for Research in Mathematical Sciences Video Archive, May, 2019. See http://www.fields.utoronto.ca/video-archive/event/2774
- Scott Aaronson on "Computational complexity and fundamental physics", 2016, https://www.youtube.com/watch?v=XsHxtcgxO1w
- William Cook: "Information, Computation, Optimization: Connecting the Dots in the Traveling Salesman Problem", 2018, https://www.youtube.com/watch?v=q8nQTNvCrjE&t=35s and http://www.math.uwaterloo.ca/tsp/index.html
- In Our Time – Unsolved Problems in Computer Science. BBC Radio 4. https://www.bbc.co.uk/programmes/b006qykl/topics/Unsolved_problems_in_computer_science

University Policies

E-mail Communication
As per university regulations, all students are required to check their <mail.uoguelph.ca> e-mail account regularly: e-mail is the official route of communication between the University and its students.

When You Cannot Meet a Course Requirement
When you find yourself unable to meet an in-course requirement because of illness or compassionate reasons, please advise the course instructor (or designated person, such as a teaching assistant) in writing, with your name, id#, and e-mail contact. See the undergraduate calendar for information on regulations and procedures for Academic Consideration.

Drop Date
Students will have until the last day of classes to drop courses without academic penalty. The deadline to drop two-semester courses will be the last day of classes in the second semester. This applies to all students (undergraduate, graduate and diploma) except for Doctor of Veterinary Medicine and Associate Diploma in Veterinary Technology (conventional and alternative delivery) students. The regulations and procedures for course registration are available in their respective Academic Calendars.

- Undergraduate Calendar - Dropping Courses https://www.uoguelph.ca/registrar/calendars/undergraduate/current/c08/c08-drop.shtml
- Graduate Calendar - Registration Changes https://www.uoguelph.ca/registrar/calendars/graduate/current/genreg/genreg-reg-regchg.shtml
- Associate Diploma Calendar - Dropping Courses https://www.uoguelph.ca/registrar/calendars/diploma/current/c08/c08-drop.shtml

Copies of out-of-class assignments
Keep paper and/or other reliable back-up copies of all out-of-class assignments: you may be asked to resubmit work at any time.

Accessibility
The University promotes the full participation of students who experience disabilities in their academic programs. To that end, the provision of academic accommodation is a shared responsibility between the University and the student. When accommodations are needed, the student is required to first register with Student Accessibility Services (SAS). Documentation to substantiate the existence of a disability is required, however, interim accommodations may be possible while that process is underway. Accommodations are available for both permanent and temporary disabilities. It should be noted that common illnesses such as a cold or the flu do not constitute a disability. Use of the SAS Exam Centre requires students to book their exams at least 7 days in advance, and not later than the 40th Class Day. More information: www.uoguelph.ca/sas

Academic Misconduct

The University of Guelph is committed to upholding the highest standards of academic integrity and it is the responsibility of all members of the University community, faculty, staff, and students to be aware of what constitutes academic misconduct and to do as much as possible to prevent academic offences from occurring. University of Guelph students have the responsibility of abiding by the University's policy on academic misconduct regardless of their location of study; faculty, staff and students have the responsibility of supporting an environment that discourages misconduct. Students need to remain aware that instructors have access to and the right to use electronic and other means of detection. Please note: Whether or not a student intended to commit academic misconduct is not relevant for a finding of guilt. Hurried or careless submission of assignments does not excuse students from responsibility for verifying the academic integrity of their work before submitting it. Students who are in any doubt as to whether an action on their part could be construed as an academic offence should consult with a faculty member or faculty advisor. The Academic Misconduct Policy is detailed in the Undergraduate Calendar: https://www.uoguelph.ca/registrar/calendars/undergraduate/current/c08/c08- amisconduct.shtml.

Recording of Materials

Presentations which are made in relation to course work—including lectures—cannot be recorded or copied without the permission of the presenter, whether the instructor, a classmate or guest lecturer. Material recorded with permission is restricted to use for that course unless further permission is granted.

Resources

The Academic Calendars are the source of information about the University of Guelph's procedures, policies and regulations that apply to undergraduate, graduate and diploma programs.

Proposed Lecture Schedule (Tentative).
Thursday September 5 • Lecture as regularly scheduled. Convex sets in R^n.
Week 1: Sept. 9-13 • Lectures as regularly scheduled. More convex sets, convex hull, polytope, Caratheodory's theorem (& proof), convex functions, minimization, Hessian, positive definite & positive semi-definite Hessian & theorems …
Week 2: Sept. 16-20 • Lectures as regularly scheduled. External representations of convex polyhedra (2 kinds), flat versus full volume polytope leading to LP formulations for later, extended formulations, Fourier-Motzkin technique and unique half-space external representation (Bronsted). Hyperplanes. Permutation matrices, double stochasticity, assignment constraints and the Birkhoff polytope.
Week 3: Sept. 23-27 • **BOTH LECTURES ARE TO BE RE-SCHEDULED** – TBA and we all have to agree. All about LP … LP, the simplex method, duality and a canonical LP inc. examples.
Week 4: Sep. 30 - Oct. 4 • Lectures as regularly scheduled. All about LP continued … Karmarkar algorithm – this is hand's on "how to" material i.e. problem modelling and problem solving & programming in Maple! Bring laptops with Maple.
Week 5: Oct. 7-11 • Lectures as regularly scheduled. Venture into graph theory. Definitions (inc. simple graph & bipartite graph) NOTE: adjacency matrix used interchangeably). Graph Isomorphism (GI), Bipartite Matching (BM), Hamilton Cycle (HC) decision problem definitions inc. "What is a decision problem?", "Big O" & algorithms. External representations (models) via extreme point formulations. Visualization of graphs in Maple and their adjacency matrices.
Week 6: Oct. 16-18 • * HOLIDAY * Monday October 14 & Tuesday October 15. • **THURSDAY LECTURE IS TO BE RE-SCHEDULED** – TBA and we all have to agree. More on models of BM, GI and HC. The TSP, TSP polytope and HC. Time and space complexity, verification of a solution e.g. certificate versus finding a solution, polynomial time. Contrast these models and solution techniques e.g. BM/GI/HC. Permutation matrices AND P_{n-1} and HC_n.
Week 7: Oct. 21-25 • **TUESDAY LECTURE IS TO BE RE-SCHEDULED** – TBA and we all have to agree. Reduction and completeness and NP-completeness – HC and SGI via $P\mathbf{G}P^t=\mathbf{H}$. FYI, Stephen Cook is next door at UToronto. See. http://www.cs.toronto.edu/~sacook/..
Week 8: Oct. 28 - Nov. 1 • Lectures as regularly scheduled. More on a linearized version (polynomial equivalency) of HC and SGI via $P\mathbf{G}P^t=\mathbf{H}$. e.g. Q matrices (Matt), Q polytope and Q-polytope series.
Week 9: Nov. 4-8 • Lectures as regularly scheduled. The real deal – polynomial time decision problems versus NP-complete decision problems. Review of the World of Complexity Theory Intro – diagram, P, NP, NP-complete, BM, GI, HC, TSP and all co-problems too. Introduction to the **P**=?**NP** conundrum. Please read https://www.win.tue.nl/~gwoegi/P-versus-NP/sipser.pdf . THE current issue in discrete mathematics, the dependency of world communications, secrecy, security. Hopefully this motivates each of you to get-out and pay attention to spook journals, quality graph theory and discrete mathematics journals and the WHY behind weak public key crypto AND where you can make a contribution. Summary of the history of the development of complexity theory
Week 10: Nov. 11-15 • Lectures as regularly scheduled. Watch together – selected pieces with commentary by me. Watch https://www.youtube.com/watch?v=q8nQTNvCrjE&t=35s … FYI William Cook is next door at UWaterloo. See https://www.math.uwaterloo.ca/~bico/
Week 11: Nov. 18-22 • Lectures as regularly scheduled. Reserved times for class presentations.
Week 12: Nov. 25-29 • Lectures as regularly scheduled. Extended material and Q/A arising from class presentations.