Non-commutative measure theory and function theory

Robert T.W. Martin
University of Manitoba

Measure theory on the complex unit circle and analytic function theory in the unit disk, in particular the theory of Hardy spaces, are fundamentally connected. Several celebrated theorems due to P. Fatou, G. Herglotz, F. and M. Riesz and G. Szegö describe the relationship between these theories.

We will show that many of these classical results have natural extensions to the multivariate and non-commutative settings of the *full Fock space*, or *free Hardy space* of square–summable power series in several non-commuting variables and *positive non-commutative (NC) measures*. Here a positive NC measure is any positive linear functional on the *free disk system*, the self-adjoint subspace of operators generated by all NC monomials in the left free shifts, which act as left multiplication by the independent NC variables on the free Hardy space. Our generalizations of the single–variable theory typically require new operator–theoretic techniques and recover the classical results, with new proofs, as a special case.