Open economy neoclassical growth models and the role of life expectancy

Tselmuun Tserenkhuu[a b], Stephen Kosempel[a]

[a] *Department of Economics & Finance, University of Guelph, Guelph, ON N1G 2W1, Canada*

Abstract

This paper applies the Ramsey-Cass-Koopmans (RCK) growth model to an open economy so that, when calibrated with standard parameter values that are commonly used in the small open economy macroeconomic literature, the time paths of the model variables and the speeds of convergence implied by the model conform with empirical evidence. Open-economy versions of the RCK growth model lead to several counterfactual conclusions including: infinite speeds of convergence for physical capital and output; and unbalanced consumption and asset growth. We avoid these undesired results by extending the baseline model with human capital, international credit constraints, and finite horizons. Given its finite-horizons feature, our model allows us to study the growth implications of changes in life expectancy from the perspective of an open economy, as most of the existing theoretical-quantitative literature that focus on the relationship assumes a closed economy. We find that increased life expectancy has positive but diminishing marginal effect on long-run output per effective labor. Using our model, we quantify the contribution of life expectancy to the long-run economic performance of Canada, sub-Saharan Africa, and the Organisation for Economic Co-operation and Development (OECD) member countries over the past six decades.

Keywords: Speed of convergence, Small open economy, Finite horizons, Life expectancy

JEL classification: O40, E10, F41, J11

[b]Corresponding author: Tselmuun Tserenkhuu, E-mail: ttserenk@uoguelph.ca
Stephen Kosempel, E-mail: kosempel@uoguelph.ca

1. Introduction

It is well known that the standard closed-economy neoclassical growth framework has rich transitional dynamics and provides an accurate description of various aspects of economic development. As pointed out in Barro and Sala-i Martin (2004), if economies are closed and capital is viewed broadly to include both physical and human capital, its predictions regarding speeds of convergence and the time paths of the key model variables accord well with empirical evidence. However, given the substantial borrowing and lending and the trade of goods and services that exist across borders of countries around the world, the widely used closed economy assumption is difficult to justify, especially if the domestic economy's growth path is likely to be affected by its openness to the world. Using data from the World Development Indicators by the World Bank, Table 1 below reports the OLS estimates from the simple linear regression between openness—proxied by the average trade share between 1990-2018—and the natural logarithm of average per capita gross domestic product (GDP) for the same period, for three different groups of countries: a sample of 158 countries that constitute the World group, a sample of 37 countries that are members of the OECD, and a sample of 39 countries in sub-Saharan Africa. The trade share is measured by exports plus imports as a percentage of GDP and the per capita GDP is measured in purchasing power parity (PPP) constant 2017 international dollars. For each group, the average trade share is found to have a positive and statistically significant effect on average GDP per capita. Given the obvious endogeneity and omitted variable issues, the results are interpreted as indication of strong correlation between the two variables. Nevertheless, we note the existence of rich empirical literature that supports the casual positive effect of trade openness on economic growth, including Edwards (1998), Frankel and Romer (1999), Dollar and Kraay (2004), Lee et al. (2004), Freund and Bolaky (2008), Chang et al. (2009), and Brueckner and Lederman (2015), among others.

Although the observed relationship between openness and growth supports using an open economy framework for economic analysis, when extended to a small open economy that can borrow and lend freely in international capital markets at a given real interest rate, the baseline model of Ramsey (1928), Cass (1965), and Koopmans (1965)—henceforth RCK—leads to several counterfactual predictions, namely, infinite speeds of convergence for physical capital and output; and

Table 1: Trade openness and GDP per capita

	Dependent variable: ln(Average GDP per capita)		
	World	The OECD	Sub-Saharan Africa
Average trade share	0.004^{***}	0.0028^{*}	0.0085^{***}
	(0.0008)	(0.0014)	(0.0017)
constant	3.64^{***}	10.18^{***}	2.88^{***}
	(0.078)	(0.14)	(0.12)
Adj R-squared	0.14	0.07	0.39

Notes: Standard errors are in brackets. $^{***}p < .01, ^{**}p < .05, ^{*}p < .1$.

unbalanced consumption and asset growth.

The objective of the current paper is twofold. First, our main objective is to apply the RCK growth model to the case of a small open economy while avoiding the foregoing problematic results. Specifically, we extend the baseline RCK growth model with human capital and international credit constraints, in the spirit of Barro et al. (1995), and with finite horizons, in the spirit of Blanchard (1985). The former two features work to slow down convergence, while the latter one is for correcting the long-run behaviour of consumption and assets. Our model features diminishing returns with respect to human capital and the transitional dynamics involves a gradual increase in human capital from its initial value to its steady state value. Given the constraint of domestic saving on the accumulation of human capital and the complementarity between different types of capital in the production function, physical capital also rises gradually toward its steady-state value despite the fact that foreign financing makes it easy to accumulate it rapidly. This implies that the convergence speeds for capital stocks and output will no longer be infinite. Moreover, adjustments in the domestic economy's effective time-preference rate, which arise out of the finite horizon feature, ensure that both consumption and assets experience balanced growth in the long-run. In fact, we find that, when calibrated with standard parameter values that are commonly used in the small open economy macroeconomic literature, the time paths of the model variables and the speeds of convergence implied by our model are consistent with observed empirical evidence.

Given the finite-horizons feature of the model and that there exist considerable variations in life expectancy across regions around the world, our second objective is, using our framework, to study the implications of changes in life expectancy on the steady state and transitional dynamics of the domestic economy. Most of the existing theoretical-quantitative literature that focus on the relationship between life expectancy and growth assumes a closed economy. We contribute to this literature by analyzing the relationship in the context of a small open economy. In our model, increased life expectancy raises human capital investment, which is in line with standard theory of human capital. Moreover, followed from its feature of diminishing returns to human capital, our model predicts that increased life expectancy has positive but diminishing marginal effect on long-run output per effective labor.

The variations in life expectancy across regions can be seen from Figure 1 below. Although the average life expectancy has an increasing trend for most parts of the world, there is a significant and persistent gap between the OECD group and the sub-Saharan African group over the years. For example, according to the latest available data, in 2018, the average life expectancy at birth for the OECD countries was 80.6 years, whereas this figure was only 62.3 years for the sub-Saharan African countries. We can see that, among the OECD countries, life expectancy in Canada has consistently been above the group's average for the period between 1960 and 2018. Figure 1 also shows that the gap in life expectancy levels between the OECD and sub-Saharan African countries has narrowed very slowly (by only 5 years) over the last half century. In our numerical analysis, we consider the long-run economic outcome of sub-Saharan Africa if its average life expectancy at birth converges from its 2018 level to the level in their OECD counterparts. We also quantify the contribution of life expectancy to the long-run economic performance of the two regions as well as Canada over the past six decades. Within the OECD, we consider Canada specifically because it has been used frequently in the macroeconomics literature as an example of a small open economy, including Mendoza (1991), Schmitt-Grohe (1998), Blankenau et al. (2001), Schmitt-Grohe and Uribe (2003), Letendre (2004), and Letendre and Luo (2007).

The rest of this paper is organized as follows: Section 2 covers the related literature; Section 3 introduces the model; Section 4 provides some numerical results; and Section 5 concludes.

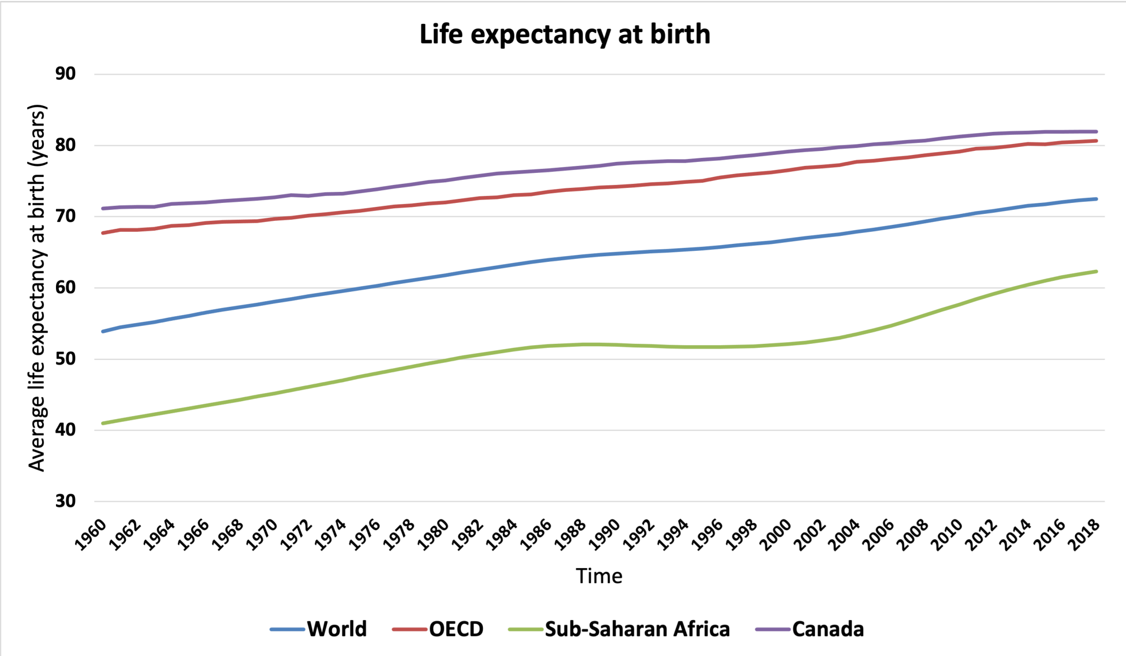

Figure 1: Average life expectancy at birth. Life expectancy at birth is defined as the average number of years a newborn is expected to live if mortality patterns at the time of its birth remain constant throughout its life. Data are from the World Development Indicators by the World Bank.

2. Related literature

In this section we review two strands of literature that are related to the current paper. First, our work relates to the theoretical papers that focus on the dynamics of a small open economy. Second, we relate to the theoretical literature that focuses on quantifying the impact of life expectancy on growth.

2.1. The dynamics of a small open economy

The first line of research that relates to our work considers the dynamics of a small open economy. In that this paper applies the RCK growth model to an open economy, it is closely related to Barro et al. (1995). Barro et al. (1995) show that the baseline open-economy RCK model extended with accumulation of human capital and international credit constraints can account for the observed gradual convergence of income per capita. In deriving their results, the authors consider a restricted case where the world interest rate is equal to the time-preference rate of the economy. That is, the world interest rate is set equal the steady-state interest rate that would apply if the domestic economy were closed. However, as shown in Barro and Sala-i Martin (2004), if the world interest rate is different from the time-preference rate, there exists no stationary equilibrium for the model. To avoid this situation, the authors suggest to introduce some mechanisms—finite horizons or a variable time-preference rate—into the model that can eliminate any gap between the two rates. In this paper, we extend Barro et al.'s (1995) analysis with finite horizons to derive the stationary equilibrium in which both consumption and assets per effective labor converge to positive constants.

A variable time-preference rate is considered in the works of Mendoza (1991), Atkeson and Kehoe (2000), and Kawagishi and Mino (2016). Mendoza (1991) extends the real-business-cycle (RBC) model to the case of a small open economy and improves its predictions from the business cycle perspective. Specifically, his model can account for the observed positive saving-investment correlation and the countercyclical behavior of the balance of trade. The author shows that, due to the separation of saving and investment that characterizes a small open economy, combined with the absence of capital-adjustment costs, the basic real-business-cycle model in a small open economy performs poorly in terms of mimicking the observed business-cycle statistics of investment. In the open economy framework, when the productivity shock hits the economy, the capital stock is rapidly and freely adjusted to maintain equality of the expected marginal returns of domestic capital and foreign assets.

To moderate the variability of capital accumulation and thus investment, rather than relying on imperfections in international capital mobility as in the current paper, Mendoza (1991) embeds capital-adjustment costs into the model and assumes perfect capital mobility. In addition, in the spirit of Uzawa (1968) and Obstfeld (1981), an endogenous rate of time preference—captured in the proposed stationary cardinal utility function—that increases with past consumption levels is introduced into his model to ensure a well-defined stationary equilibrium for the holdings of foreign assets. Given this preference formulation, as mentioned in the paper, the dynamics of the model works as follows: as long as the time-preference rate is smaller (greater) than the interest rate, households choose to accumulate (deplete) foreign assets in order to fund an increasing (decreasing) consumption. Once the rate of time preference adjusts to the world's real interest rate, the accumulation of external assets reaches its steady state level. The main attraction of our modelling approach in comparison to Mendoza (1991) is that, in our framework, the horizon index can be chosen arbitrarily anywhere between zero and infinity and this allows us to study the changes in life expectancy on the behavior of the economy. In Mendoza's (1991) analysis, in contrast, the individuals are assumed to be infinitely lived.

More recently, Kawagishi and Mino (2016) study the implications of the variable time preference rate on cross-country income convergence using the standard Heckscher-Ohlin model. In their model, the small open economy is assumed to have a smaller capital stock than the rest of the world—which is assumed to have reached its steady-state equilibrium already. As for the time preference rate of the households, they consider two different specifications, both of which are endogenously determined: (i) the time preference rate increases with consumption—the hypothesis initiated by Uzawa (1968); and (ii) the time preference rate decreases with consumption.

Unlike the results obtained in Atkeson and Kehoe (2000), who consider the case of a fixed rate of time preference in the same framework and find that the small open economy can never catch up with the rest of the world, Kawagishi and Mino (2016) conclude that the small country can catch up with the rest of the world when the time preference rate increases with consumption. The intuition of their model is given as follows. If the discount rate is an increasing function of consumption, the individuals are more patient when their consumption level is low. Therefore, the small country that starts with a low level of capital stock will attain a higher level of savings, which in turn fosters its capital accumulation. Consequently, the small open economy converges to the same steady-state level of income as the rest of the world. If the discount rate is a decreasing function of consumption, on the other hand, the small country is less patient than the rest of the world and it fails to accumulate high enough capital stock to reach the steady-state level of income that the rest of the world has already achieved.

By construction the Heckscher-Ohlin model—on which both of Atkeson and Kehoe (2000) and Kawagishi and Mino's (2016) analyses are based—is a general equilibrium trade model of two countries that have varying specialties. In contrast, in our framework, following Barro et al. (1995) and Barro and Sala-i Martin (2004), the only function of trade is to allow the small open economy's production to diverge from its expenditure on consumption and investment. In other words, we abstract from patterns of specialization in production and focus on the intertemporal aspects of international trade. Furthermore, in both Atkeson and Kehoe (2000) and Kawagishi and Mino (2016), the authors consider income convergence between a small open economy—that has a lower capital stock, and thus has a different specialization, than the rest of the world—and the rest of the world. In contrast, in the current paper, we consider equilibrium dynamics in a world where the small open economy and the rest of the world are identical and we focus on the small open economy's transitional dynamics and speed of convergence towards its own steady state.

2.2. The impact of life expectancy on economic growth

In the second line of research that relates to our work, life expectancy is either modelled as an exogenously given parameter or it is determined endogenously in the model. The existing models of endogenous life expectancy, where the survival probability is dependent on a choice variable within the model, are often based on a two or three period overlapping-generations model and usually generate multiple steady states that correspond to different development regimes. In general, the main focus of these studies is to model the link between demographic transition and economic outcomes that captures the observed experience of both poor and rich economies. Furthermore, the models can explain why some countries may permanently lag behind others and be trapped in a persistent low level of income per capita. Papers that feature endogenous life expectancy include, among others, Kalemli-Ozcan (2002), Blackburn and Cipriani (2002), Chakraborty (2004), and Cervellati and Sunde (2005).

Studies in which life expectancy is exogenous are often based on Blanchard's (1985) perpetual youth model where agents face, throughout their life, a constant probability of death per unit of time. This approach is appealing in that, given the assumption of constant probability of death, it solves the aggregation problem that is inherent in an economy with finitely lived agents and allows one to quantify the direct effects of changes in life expectancy on the behavior of the economy. The current paper and the papers discussed below utilize this approach. Moreover, our model is similar to the ones discussed below[1] in that increased life expectancy has positive effect on output by making investments in human capital more profitable and thus encouraging its accumulation.

De la Croix and Licandro (1999) study the effect of life expectancy on growth in the overlapping-generations framework of Blanchard (1985), extended with schooling decisions and a human capital

[1] The exception is Prettner (2013). He considers the effects of aging on purposeful R&D investments and thus the transmission channel of the impact of life expectancy on growth is fundamentally different from ours.

externality. The model has endogenous growth in the production side where human capital is the only input. In their model, an increase in life expectancy affects the growth rate through three channels: (i) agents die later on average, thus the depreciation rate of aggregate human capital decreases and the growth rate increases; (ii) agents spend more time on schooling because the expected flow of future wages has risen, and the human capital per capita and thus the growth rate increases; and (iii) the economy consists of more of old agents who did their schooling a long time ago and this influences the growth rate negatively. Thus, depending on which channels dominate, the total effect of an increase in life expectancy can be either negative or positive. Their numerical results show that the effect of life expectancy on growth is hump shaped. When life expectancy is below a certain threshold, the first two effects dominate and thus its effect on the growth rate is positive. However, after some point, the negative effect starts to offset the positive effects and an exogenous rise in life expectancy leads to a drop in the growth rate. From an empirical point of view, the authors suggest that the effect of increased life expectancy on growth is positive for countries with a relatively low life expectancy, but could be negative for more advanced economies.

Based on Blanchard (1985), Kalemli-Ozcan et al. (2000) develop a perpetual youth model with endogenous schooling decisions and study the relationship between exogenous mortality decline and human capital accumulation in both partial and general equilibrium settings, focusing on the steady state. The model incorporates both physical and human capital as inputs into production. In their model, a decline in mortality increases the horizon over which investments in schooling will be paid off. This results in an increase in the accumulation of human capital, which in turn increases output. Their calibration results suggest that, when life expectancy increases from 33 to 83, the length of schooling increases from 10.69 to 23.51.

Both De la Croix and Licandro (1999) and Kalemli-Ozcan et al. (2000) assume that agents have to choose the time allocated to schooling before starting to work and thus can only accumulate human capital before joining labor force. Different from theirs, in our paper, we assume that agents can accumulate human capital in each period of their life. In other words, we allow for the human capital accumulation that can be obtained through on-the-job training while agents work.

Hu (1999) extends Lucas's (1988) endogenous growth framework with finite horizons in the spirit of Blanchard (1985) and studies the effect population aging on long-run economic growth.

In his model, depending on the effect of higher growth rate on the steady state ratio of aggregate consumption and physical capital, there may exist multiple steady-state equilibria. According to his numerical results, population aging that takes the form of a lower probability of death combined with a slowdown in population growth is found to have an insignificant effect on the growth rate of the high-growth equilibrium, while at the low-growth equilibrium, population aging raises the growth rate of output.

Prettner (2013) studies the effects of population aging on long-run economic growth by setting up an endogenous growth model that features R&D sector with knowledge spillovers and endogenous fertility decisions. His model features finite individual planning horizons in the spirit of Blanchard (1985). He finds that, in general, decreases in fertility negatively affect long-run growth, while decreasing mortality has positive effect on growth. When the knowledge spillovers are strong and the population size is constant, the positive effects of decreasing mortality are found to overcompensate for the negative effects of decreasing fertility and thus population aging fosters long-run growth. However, in the case with weaker spillovers and a growing population size, its effect depends on the relative changes between fertility and mortality.

Different from De la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Hu (1999), and Prettner (2013)—all of whom consider the implications of life expectancy on long-run growth rate—in our analysis, we pay particular attention to the transitional dynamics of the economy and abstract from endogenous growth. Moreover, all the above-mentioned papers are based on a closed economy assumption[2] where factor prices are determined in the aggregate factor markets. In contrast, we quantify the effects of changes in life expectancy on the behavior of the domestic economy in the context of a small open economy where the real interest rate is fixed at the word rate.

[2]Heijdra and Romp (2009) study the effects of demographic shocks on the macroeconomic performance of an industrialized small open economy by extending the perpetual youth framework of Blanchard (1985) with an age-dependent mortality rate. However, different from ours, as in De la Croix and Licandro (1999) and Kalemli-Ozcan et al. (2000), in their model, agents can only accumulate human capital before joining labor force. Moreover, unlike ours, their model features endogenous growth in the spirit of Lucas (1988).

3. The Model

3.1. The counterfactual predictions of the baseline open-economy RCK model

Before formally introducing our extended model, in this subsection, closely following Barro and Sala-i Martin (2004), we discuss the existence of the problematic conclusions of the baseline open-economy RCK model, mentioned in the Introduction, and explain how our model solves them.

As demonstrated in Barro and Sala-i Martin (2004), in the baseline open-economy RCK model where the real interest rate is fixed at the world rate, the equality between marginal productivity and rental price of capital alone determines capital intensity and thus investment of the domestic economy. Consequently, the capital stock and investment per effective labor instantly jump, by means of international capital flows, to their respective constant steady state values. Moreover, the constancy of capital stock means that the output and wage rate per effective labor will also be constants. These implied infinite speeds of convergence for capital, output, and the wage rate are some of the problematic implications of the framework.

The time paths of the small open economy's consumption and assets are shown to behave differently depending on the relationship between the rate of time preference of the economy and the exogenously given world real interest rate it faces. If the time preference rate is greater than the interest rate, the domestic economy is relatively impatient and it borrows to enjoy a high level of consumption early on, with the trade-off of having low consumption growth later on. As a result, consumption tends to zero in the long-run. Given this consumption path, assets in the economy approach a negative number asymptotically. That is, an impatient country ends up mortgaging all its capital and labor income.

If the time preference rate is lower than the interest rate, on the other hand, the domestic economy is relatively patient and we get the opposite results: consumption and assets would approach infinity—a result that violates the existence of small open economies. Thus, unless the domestic economy's time preference rate and the world interest rate are preset to be equal—in this case there exist a steady-state equilibrium in which both consumption and assets per effective labor tend to positive constants as in the baseline closed-economy RCK model—the baseline open-economy RCK model leads to inaccurate predictions regarding the behavior of consumption and assets.

In an attempt to improve the poor performance of the framework, Barro and Sala-i Martin (2004) consider the following extensions in turn: a constraint on international borrowing, adjustment costs in the accumulation of human capital, variations in preference parameters, and finite horizons. However, considered separately, these extensions are shown to solve some, but not all, of the problematic predictions of the baseline open-economy RCK model mentioned above. In particular, the former two features each work to slow down the instantaneous adjustment of capital stock, whereas the latter two are for eliminating the gap between the interest rate and the time-preference rate that the economy faces.

To apply the RCK model to a small open economy in a fully satisfactory way, we modify the baseline RCK growth model to incorporate human capital, international credit constraints, and finite horizons. Specifically, the former two features work to slow down convergence, while the latter one is for correcting the long-run behaviour of consumption and assets. The combined effects from these extensions will allow the model to produce the desired results, thereby providing an improved framework for macroeconomic analysis.

3.2. The structure of the model

We extend the baseline RCK model of exogenous growth with finite horizons, human capital, and borrowing constraint in a small open economy environment. Based on the perpetual youth model by Blanchard (1985), we incorporate finite horizons in our model by assuming that agents face, throughout their life, a time invariant probability of death per unit of time, which is captured by λ. The assumption that the probability of death is invariant with age can be unrealistic; however, it solves the aggregation problem that is inherent in an economy with finitely lived agents and allows us to study the direct effects of changes in life expectancy on the behavior of the economy. Given this extension, although the preference parameters are constant for individuals, the aggregation over agents who differ with respect to consumption and assets leads to a result in which the domestic economy's effective time-preference rate is an increasing function of the ratio of human capital and consumption per effective labor. Moreover, in equilibrium, this ratio adjusts so that there is no gap between the effective time-preference rate and the rate of return that the economy faces on its assets. As a result, consumption and assets will remain positive.

We assume that the population grows at a constant rate of n. At any period t, a large cohort, whose size is $(\lambda + n)e^{nt}$, is born. Thus, a size at date t of a cohort born at date j will be $(\lambda + n)e^{-\lambda t+(\lambda+n)j}$ and the population size at date t will be e^{nt}. Although each agent is uncertain about the time of death, with large cohorts, the size of a cohort declines nonstochastically through time. The probability that a person born at date j dies during period t is $\lambda e^{-\lambda(t-j)}$. The life expectancy for someone born at time 0 will be $\int_0^\infty (t-j)\lambda e^{-\lambda(t-j)}d(t-j) = 1/\lambda$. Agents are born with no wealth and have no bequest motives. In each period, agents are endowed with one unit of time that they can allocate to labor inelastically.

As in Yaari (1965) and Blanchard (1985), uncertainty about death and the absence of a bequest motive create the need for a life insurance market. We assume that there exist life insurance companies and agents can contract to make (or receive) a payment contingent on their death (or survival). Furthermore, we assume that the insurance market is competitive so that the companies charge an actuarially fair premium (which is equal to λ). Thus, an agent with asset holdings of a will receive λa from the insurance company if she survives and pay a if she dies. There are three types of assets in the economy: physical capital, human capital, and a net foreign financial asset. We assume that, when an agent dies, she leaves behind her physical capital and net foreign asset holdings to be collected by the insurance company, whereas the holdings of human capital are terminated with the agent. Thus, the holdings of assets that are acceptable by the insurance company—denoted by a—include physical capital and net foreign assets only.

3.3. Household's problem

Denote consumption, total assets that are acceptable by the insurance company, net foreign assets, physical capital, and human capital holdings of an agent born at time j as of time v as $c(j,v)$, $a(j,v)$, $f(j,v)$, $k(j,v)$, and $h(j,v)$, respectively. Assuming a general isoelastic functional form for the instantaneous utility, the dynamic optimization problem for a representative agent of cohort j at date t is given by:

$$max \int_t^\infty \left[\frac{c(j,v)^{1-\theta} - 1}{1-\theta}\right] e^{-(p+\lambda)(v-t)}dv \tag{1}$$

$$s.t. : \dot{a}(j,v) + \dot{h}(j,v) = [r(v) + \lambda]a(j,v) + [q(v) - \delta_h]h(j,v) + w(v) - c(j,v) \tag{2}$$

Here p, θ^{-1}, $r(v)$, $q(v)$, and δ_h correspond to the pure rate of time preference, the elasticity of intertemporal substitution, the real interest rate at time v, the rental rate of human capital at time v, and the depreciation rate for an individual's human capital[3], respectively. We note that, in the above expected utility formulation in (1), $p + \lambda$ captures the effective rate of time preference of an agent who faces uncertainty about the time of her death. We also assume that labor productivity is independent of age, so that the wage rate $w(v)$ is the same for all j.

An international borrowing constraint is introduced into the model, following Barro et al. (1995), by assuming that only physical capital can act as collateral on foreign loans, whereas human capital and raw labor cannot. In other words, the foreign debt can be positive but cannot exceed the quantity of physical capital:

$$-f(j, v) \leq k(j, v) \tag{3}$$

Initially, we restrict our attention to the case where the borrowing constraint is binding; since, otherwise, the economy jumps instantly to its steady state as in the baseline open-economy RCK model. In our model, the presence of both types of capital and the binding borrowing constraint will slow down the infinite speeds of convergence of capital and output. Given the binding borrowing constraint, we must have that:

$$-f(j, v) = k(j, v) \tag{4}$$

$$\implies a(j, v) = 0 \quad \forall v \tag{5}$$

Under certain conditions the equality restriction in (4) will eventually fail to hold, which means that at some point the economy may acquire equity. These conditions will be identified when the dynamics of the model are discussed in Section 3.6.

Next, from (5) the flow budget constraint in (2) can be written as follows:

$$\dot{h}(j, v) = [q(v) - \delta_h]h(j, v) + w(v) - c(j, v) \tag{6}$$

[3]Knowledge and skills may depreciate for various reasons, including obsolescence, mental and physical decline due to aging or injury, or a period of labor market inactivity (unemployment). Although we do not formally model these things, incorporating a depreciation rate for human capital into the model is a way to account for them.

The agent maximizes (1) subject to (6), taking $w(v)$ and $q(v)$ as given. The first-order conditions for the dynamic optimization problem imply:

$$\frac{\dot{c(j,v)}}{c(j,v)} = \frac{1}{\theta}\left[q(v) - \delta_h - p - \lambda\right] \tag{7}$$

The transversality condition is given by:

$$lim_{v\to\infty}[Q(t,v) \cdot h(j,v)] = 0 \tag{8}$$

where $Q(t,v) \equiv exp\{\int_t^v -[q(u) - \delta_h]du\}$. By integrating (7) and imposing the transversality condition in (8), we can derive the agent's lifetime budget constraint as follows:

$$\int_t^\infty c(j,v)Q(t,v)dv = h(j,t) + \widetilde{w}(t) \tag{9}$$

where $\widetilde{w}(t)$ is the present value at date t of agent's lifetime wage income. Using equations (7) and (9), we can express consumption as a function of wealth:

$$c(j,t) = \Delta(t)^{-1}\left[h(j,t) + \widetilde{w}(t)\right] \tag{10}$$

where $\Delta(t)^{-1}$ denotes the propensity to consume out of wealth and $\Delta(t)$ is given by:

$$\Delta(t) = \int_t^\infty exp\left\{\theta^{-1}\int_t^v \left[(1-\theta)(q(u)-\delta_h) - (\rho+\lambda)\right]du\right\}dv \tag{11}$$

or

$$\frac{\dot{\Delta}(t)}{\Delta(t)} = -\Delta^{-1}(t) - \theta^{-1}\left[(1-\theta)(q(t)-\delta_h) - (p+\lambda)\right] \tag{12}$$

3.4. Firms

Each firm in the perfectly competitive market has access to the following production technology:

$$Y(t) = F(K(t), H(t), L(t), Z(t)) = [Z(t)L(t)]^{1-\alpha-\eta} K(t)^{\alpha} H(t)^{\eta} \tag{13}$$

where $Y(t)$ is the aggregate output, $K(t)$ is the aggregate physical capital, $H(t)$ is the aggregate human capital, $L(t)$ is the aggregate labor input, and $Z(t)$ is the level of technology, which is assumed to grow at a constant rate of x, in the economy at time t. We assume that the function $F(\cdot)$ is Cobb-Douglas and satisfies the usual neoclassical properties. The price of the final good is normalized to one. Moreover, for simplicity, the initial values of Z and L are both assumed to be one. Following the standard use of notations, we use lowercase letters with hats ($\wedge$) to define variables in effective labor units. Equation (13) can then be expressed in intensive form as follows:

$$\hat{y}(t) = \hat{k}(t)^{\alpha} \hat{h}(t)^{\eta} \tag{14}$$

Factor prices can be found from the representative firm's profit maximization problem as follows, with δ_K denoting the depreciation rate of physical capital:

$$r(t) = MP_K(t) - \delta_K = \alpha \hat{y}(t)/\hat{k}(t) - \delta_K \tag{15}$$

$$q(t) = MP_H(t) = \eta \hat{y}(t)/\hat{h}(t) \tag{16}$$

$$w(t) = MP_L(t) = (1 - \alpha - \eta)\hat{y}(t)Z(t) \tag{17}$$

Given the assumption of a small open economy, the domestic accumulation of assets and capital stocks have no effect on the path of the world interest rate and for the domestic economy $r(t)$ will be treated as exogenous. For simplicity, we assume that the world real interest rate is fixed at r. Then from (15) we derive:

$$\hat{k}(t) = \left[\frac{\alpha}{r + \delta_K}\right] \hat{y}(t) \tag{18}$$

If there exists no human capital—i.e. $\eta = 0$ and there exists only physical capital—in the economy, then given r and (14), equation (15) alone determines the capital per effective labor, which will be a constant. In other words, the capital intensity of the economy instantly converges to its steady state level. The introduction of the both types capital in the production function allows us to avoid this result.

Combining (14) and (18), we can express output per effective labor in terms of human capital per effective labor only:

$$\hat{y}(t) = \left[\frac{\alpha}{r + \delta_K}\right]^{\frac{\alpha}{1-\alpha}} \hat{h}(t)^{\frac{\eta}{1-\alpha}} = B\hat{h}(t)^{\varepsilon} \tag{19}$$

where $B \equiv \left[\frac{\alpha}{r+\delta_K}\right]^{\frac{\alpha}{1-\alpha}}$ and $\varepsilon \equiv \frac{\eta}{1-\alpha}$, which corresponds to the total capital share. Note that $0 < \alpha + \eta < 1$ implies $0 < \varepsilon < \alpha + \eta < 1$. Thus, from (19), we can see that our model features diminishing returns with respect to human capital.

3.5. Aggregation

Following Blanchard (1985), the aggregate variables $C(t)$, $H(t)$, and $\widetilde{W}(t)$ can be derived from addition across the cohorts as follows:

$$C(t) = \int_{-\infty}^{t} c(j,t)(\lambda + n)exp\{-\lambda t + (\lambda + n)j\}dj \tag{20}$$

$$H(t) = \int_{-\infty}^{t} h(j,t)(\lambda + n)exp\{-\lambda t + (\lambda + n)j\}dj \tag{21}$$

$$\widetilde{W}(t) = \tilde{w}(t)\int_{-\infty}^{t} (\lambda + n)exp\{-\lambda t + (\lambda + n)j\}dj = \widetilde{w}(t)e^{nt} \tag{22}$$

By aggregating (10):

$$C(t) = \Delta^{-1}(t)\left[H(t) + \widetilde{W}(t)\right] \tag{23}$$

Using (6), (21), and the initial condition that agents are born with zero assets, $h(j, j) = 0$:

$$\dot{H}(t) = W(t) + [q(t) - \delta_h - \lambda] H(t) - C(t) \tag{24}$$

From (22):

$$\dot{\widetilde{W}}(t) = [q(t) - \delta_h + n] \widetilde{W}(t) - W(t) \tag{25}$$

where $W(t) \equiv L(t)w(t)$ is the aggregate wages at date t. Using (23)-(25) and (12), we derive the change over time in aggregate consumption:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta} [q(t) - \delta_h - \rho - \lambda] - \Delta(t)^{-1}(\lambda + n)\frac{H(t)}{C(t)} + n \tag{26}$$

A comparison of equations (7) and (26) reveals that individual and aggregate consumption grow at different rates. Heijdra and Ligthart (2002) used the term Generational Turnover Effect (GTE) to explain the differences in growth rates between the individual and aggregate variables in the Blanchard model. The GTE operates as follows: At each instant a new generation is born and a cross-section of the existing population dies. Since newborn agents start with no assets (physical or human capital), their consumption is lower than average consumption. As a result, the turnover of generations drags down aggregate consumption growth.

Notice also that the laws of motion for individual and aggregate human capital accumulation, (6) and (24), differ. These equations differ in that the aggregate depreciation rate of human capital exceeds the individual level by an amount equal to the probability of dying (λ). This difference is not imposed on the model, but rather created out of its structure, and accounts for the fact that those who die have more human capital than those who are born. In other words, the depreciation rate of human capital is higher at the aggregate level due to the turnover of generations.

3.6. Transitional dynamics and the steady state

We now combine the behavior of households and firms to analyze the structure of a competitive market equilibrium. Given a fixed world real interest rate, r, the equilibrium consists of an allocation $\{c(j,t),\ h(j,t),\ k(j,t),\ f(j,t),\ a(j,t)\}$ for all living households of each cohort $j \leq t$, a production plan $\{Y(t),\ H(t),\ L(t),\ K(t)\}$ for each firm, and a set of prices $\{w(t), q(t)\}$, such that:

(i) given the interest rate and prices, the allocations for the households solve their optimization problem given by (1), (2), (4), (5), and (8);

(ii) given the interest rate and prices, the allocations for the firms solve their profit maximizing conditions given by (15)-(17);

(iii) all markets clear; below $-F(t)$ stands for the aggregate foreign debt of the domestic economy:

$$e^{nt} = L(t) \tag{27}$$

$$K(t) = -F(t) \tag{28}$$

$$C(t) + \dot{H}(t) = (1-\alpha)Y(t) - (\delta_h + \lambda)H(t). \tag{29}$$

With a given initial value of human capital per effective labor and the transversality condition, the time paths of consumption and human capital (and thus output) in effective labor units will be determined by the solution to the following dynamic system:

$$\frac{\dot{\hat{c}}(t)}{\hat{c}(t)} = \frac{1}{\theta}\left[\underbrace{\left(\eta B\hat{h}(t)^{\varepsilon-1} - \delta_h - \lambda\right)}_{(i)} - \underbrace{\left(p + \theta x + \theta(\lambda+n)\Delta(t)^{-1}\frac{\hat{h}(t)}{\hat{c}(t)}\right)}_{(ii)}\right] \tag{30}$$

$$\frac{\dot{\hat{h}}(t)}{\hat{h}(t)} = (1-\alpha)B\hat{h}(t)^{\varepsilon-1} - (\delta_h + \lambda + n + x) - \frac{\hat{c}(t)}{\hat{h}(t)} \tag{31}$$

$$\frac{\dot{\Delta}(t)}{\Delta(t)} = -\Delta^{-1}(t) + \theta^{-1}\left[(\theta-1)(\eta B\hat{h}(t)^{\varepsilon-1} - \delta_h) + p + \lambda\right] \tag{32}$$

where the Euler equation in (30) is derived from (16), (19), and (26); the resource constraint in

(31) is derived from (16), (19), and (24); and the law of motion for the inverse of the propensity to consume out of wealth in (32) is derived from (12), (16), and (19).

In order for a steady state to exist, the three variables in (30)-(32) must be constant in the long run. For consumption this can be achieved if the two terms identified on the right hand side of the Euler equation in (30) equalize. Term (i) is the rate of return to human capital, and term (ii) is the effective discount rate. Notice that both terms depend on the level of human capital. A low value of $\hat{h}$ implies a high rate of return but a low value of the effective discount rate. Then, as $\hat{h}$ rises, its rate of return falls and the effective discount rate rises. The dependency of the effective discount rate on $\frac{\hat{h}}{\hat{c}}$ is unique to the finite horizon model. In an infinite horizon model, the effective discount rate is constant at $\rho + \theta x$. The fact that the effective discount rate rises with $\hat{h}$ is important and helps to produce a long run (i.e. steady state) value for the rate of return to human capital that is above the world real interest rate, r. In fact, equations (30)-(32) above were derived under the assumption of a binding debt constraint—implied by equations (4) and (5)—and this assumption is only valid if the rate of return to $\hat{h}$ remains above r. Therefore, when analyzing the dynamics of the model, there are two cases to consider: (i) $\eta B \hat{h}^{*^{\varepsilon-1}} - \delta_h - \lambda > r$ and (ii) $\eta B \hat{h}^{*^{\varepsilon-1}} - \delta_h - \lambda = r$.

Case (i): $\eta B \hat{h}^{*^{\varepsilon-1}} - \delta_h - \lambda > r$

When the condition above holds, it implies that the borrowing constraint is binding for all time and the dynamic system (30)-(32) must be satisfied at all dates. Equations (30)-(32) form a system of three non-linear differential equations with three unknowns: $\hat{c}(t)$, $\hat{h}(t)$, and $\Delta(t)$. From the solution to the system and for a given initial value of human capital per effective labor, we can derive the time paths of consumption and human capital (and thus output, growth rate of output, physical capital, and saving rate) per effective labor. The phase diagram associated with the system is constructed in Figure 2 below. In the bottom half of the figure, the downward sloping (when $\theta > 1$) $\frac{\dot{\Delta}}{\Delta} = 0$ locus is shown in $(\hat{h}, \Delta^{-1})$ space. The $\frac{\dot{\Delta}}{\Delta} = 0$ locus divide the space into two regions and the arrows show the directions of motion in each region. In the upper half of the figure, $\frac{\dot{\hat{c}}}{\hat{c}} = \frac{\dot{\Delta}}{\Delta} = 0$ and $\frac{\dot{\hat{h}}}{\hat{h}} = 0$ loci are displayed in $(\hat{h}, \hat{c})$ space. The locus $\frac{\dot{\hat{c}}}{\hat{c}} = \frac{\dot{\Delta}}{\Delta} = 0$ is upward sloping, going through the origin and asymptotically reaching $\hat{h}^{**} = \left[\frac{\eta B}{\delta_h + \lambda + p + \theta x}\right]^{\frac{1}{1-\varepsilon}}$. At points to the left of this locus, $\hat{c}$ is rising and the arrows point upward in this region. $\hat{c}$ is falling at points to the right of the locus and thus the arrows point down in this region. The locus $\frac{\dot{\hat{h}}}{\hat{h}} = 0$ has inverse-U shape. At points above the $\frac{\dot{\hat{h}}}{\hat{h}} = 0$ locus, $\hat{h}$ is falling and the arrows point to the left. At points below this locus, $\hat{h}$ is rising and the arrows point to the right in this region. The saddle-path stable steady state—corresponds to the pair $(\hat{h}^*, \hat{c}^*)$—occurs at the intersection of the $\frac{\dot{\hat{c}}}{\hat{c}} = \frac{\dot{\Delta}}{\Delta} = 0$ and $\frac{\dot{\hat{h}}}{\hat{h}} = 0$ loci. The dynamic equilibrium in our model follows the stable saddle path indicated by the solid locus with arrows. Starting from any given initial point on this path, the economy converges towards the steady-state pair $(\hat{h}^*, \hat{c}^*)$[4]. For example, if the economy starts from an initial point on the saddle path such that $\hat{h}_0 < \hat{h}^*$, during the transition, both $\hat{h}$ and $\hat{c}$ rise monotonically along the saddle path until they converge to their respective steady state values. On the other hand, the propensity to consume out of wealth, Δ^{-1}, falls gradually along the saddle path until it reaches its steady state value, Δ^{*-1}.

In our model, the gradual increase in human capital, during the transition, from its initial value to its steady state value reflects its feature of diminishing returns. Moreover, given the constraint of domestic saving on the accumulation of human capital and the complementarity between the two types of capital in the Cobb-Douglas production function, physical capital (and thus $\hat{y}$) will also rise gradually toward its steady-state value during the transition despite the fact that foreign financing makes it easy to accumulate it quickly. This implies that the convergence speeds for capital stocks and output will no longer be infinite.

To derive the steady-state values of the model variables, we note that, in the saddle-path stable steady state, the variables in effective labor units (i.e. $\hat{h}^*$, $\hat{c}^*$, $\hat{y}^*$), the wage rate in its intensive form, $\hat{w}^*$[5], the rental rate of human capital, q^*, and the propensity to consume out of wealth, Δ^{*-1}, are all constants. The steady state ratio of consumption to human capital per effective labor, $\frac{\hat{c}^*}{\hat{h}^*}$, can be found by solving equations (30)-(32) when evaluated at the steady state. Then, given a unique positive root for $\frac{\hat{c}^*}{\hat{h}^*}$, there also exist unique steady state values for $\hat{h}^*$, $\hat{c}^*$, $\hat{y}^*$, w^*, and q^*.

[4]Along the saddle path all the first-order conditions and the transversality condition of the model are satisfied. Any paths other than the saddle path would eventually violate either the Euler equation in (30) or the transversality condition in (8).

[5]Note that $\hat{w}^* \equiv \frac{w\left(\hat{k}^*, \hat{h}^*, Z(v)\right)}{Z(v)}$.

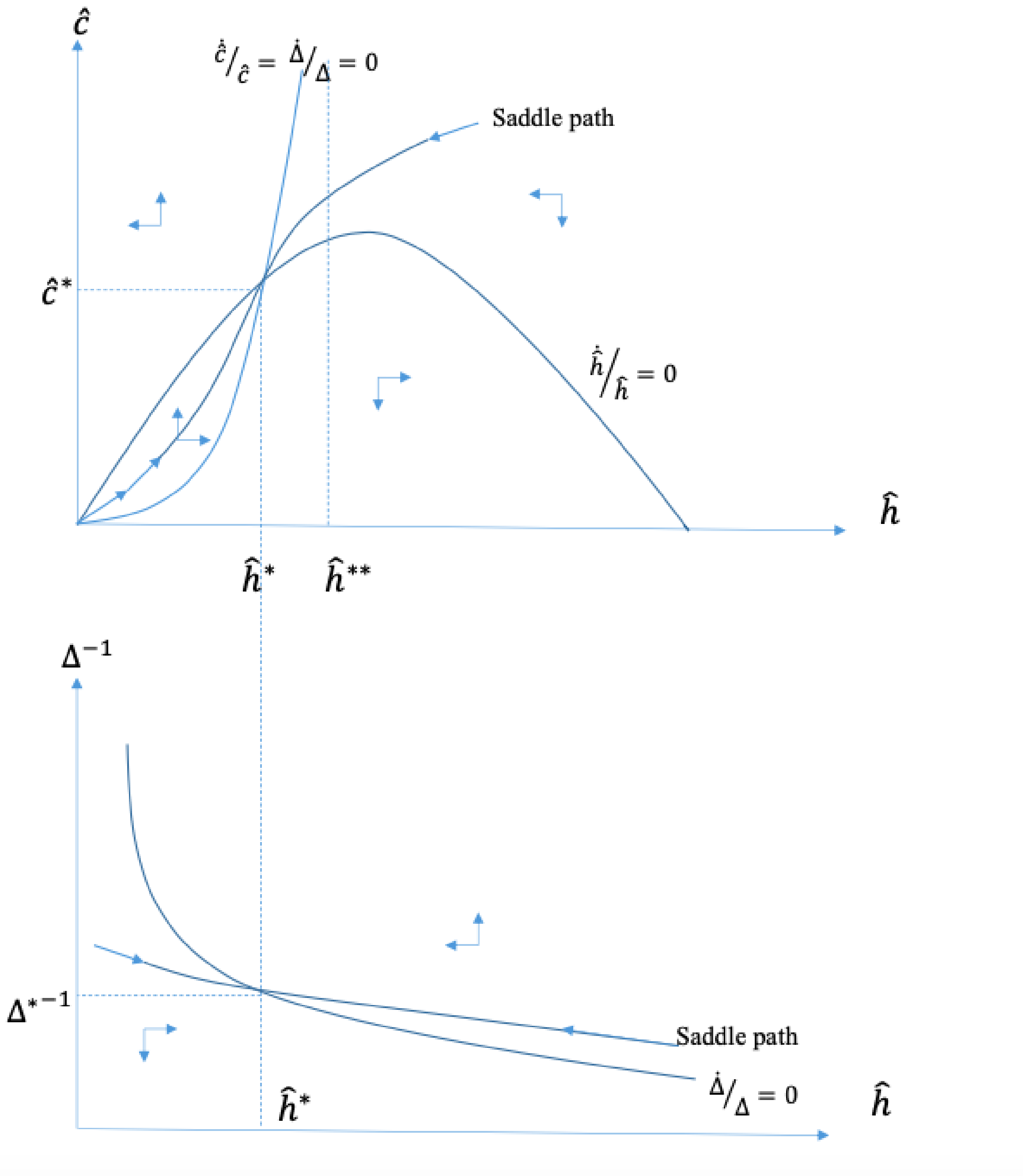

Figure 2: The phase diagram, Case (i)

In particular, given $\frac{\hat{c}^*}{\hat{h}^*}$, equation (31) at the steady state implies:

$$\hat{h}^* = \left[\frac{B(1-\alpha)}{\frac{\hat{c}^*}{\hat{h}^*} + n + x + \delta_h + \lambda} \right]^{\frac{1}{1-\varepsilon}} \tag{33}$$

Given $\frac{\hat{c}^*}{\hat{h}^*}$ and $\hat{h}^*$, we can then pin down the steady state levels of consumption and output per effective labor, and the saving rate for human capital, s^*, as follows:

$$\hat{c}^* = \left[\frac{\hat{c}^*}{\hat{h}^*} \right] \hat{h}^* \tag{34}$$

$$\hat{y}^* = B\hat{h}^{*^{\varepsilon}} \tag{35}$$

$$s^* = 1 - \frac{\hat{c}^*}{(1-\alpha)\hat{y}^*} \tag{36}$$

Case (ii): $\eta B\hat{h}^{*^{\varepsilon-1}} - \delta_h - \lambda = r$

For this case the economy starts off by following the dynamics described above until rates of return to all assets are equalized. Once the rates of return on assets are equalized, if the effective discount rate in (30) is below r, then agents will stop investing in human capital and start to accumulate foreign financial assets. In other words, the borrowing constraint will no longer hold with equality. Consequently, we will have the following flow budget constraint:

$$\dot{b}(j,v) = (r+\lambda)b(j,v) + w\left(\hat{k}^*, \hat{h}^*, Z(v)\right) - c(j,v) \tag{37}$$

Here b denotes the sum of all assets (including human capital). When agents maximize their lifetime utility in (1) subject to the above borrowing-unconstrained budget constraint in (37), the dynamic equations become:

$$\frac{\dot{\hat{c}}(t)}{\hat{c}(t)} = \frac{1}{\theta} \left\{ r - \left[\rho + \theta x + \theta(\lambda+n)\Delta(t)^{-1} \frac{\hat{b}(t)}{\hat{c}(t)} \right] \right\} \tag{38}$$

$$\frac{\dot{\hat{b}}(t)}{\hat{b}(t)} = \frac{\hat{w}^*}{\hat{b}(t)} + r - n - x - \frac{\hat{c}(t)}{\hat{b}(t)} \tag{39}$$

$$\frac{\dot{\Delta}(t)}{\Delta(t)} = -\Delta(t)^{-1} + \frac{1}{\theta}\left[(\theta-1)(r+\lambda)+\rho+\lambda\right] \tag{40}$$

Equation (40) is a first order linear differential equation, and shows that the dynamics for Δ will be independent of $\hat{c}$ and $\hat{b}$. In addition, since the last term on the right hand side of (40) is strictly positive (for $\theta > 1$), Δ will have a stable long-run value. For the other variables, the phase diagram[6] is depicted in Figure 3 below. Arrows indicate direction of movement, and (similar to the previous case) reveal that the equilibrium is a saddle point. It is worth noting that the finite horizon feature is essential for this stability because without it the effective discount rate would remain permanently below r, and both consumption and assets would grow indefinitely. In contrast, with finite horizons, the agents discount the future more heavily when their assets grow in size relative to their consumption, and this will in turn slow down the accumulation of assets, thereby creating a stable long-run outcome for both assets and consumption.

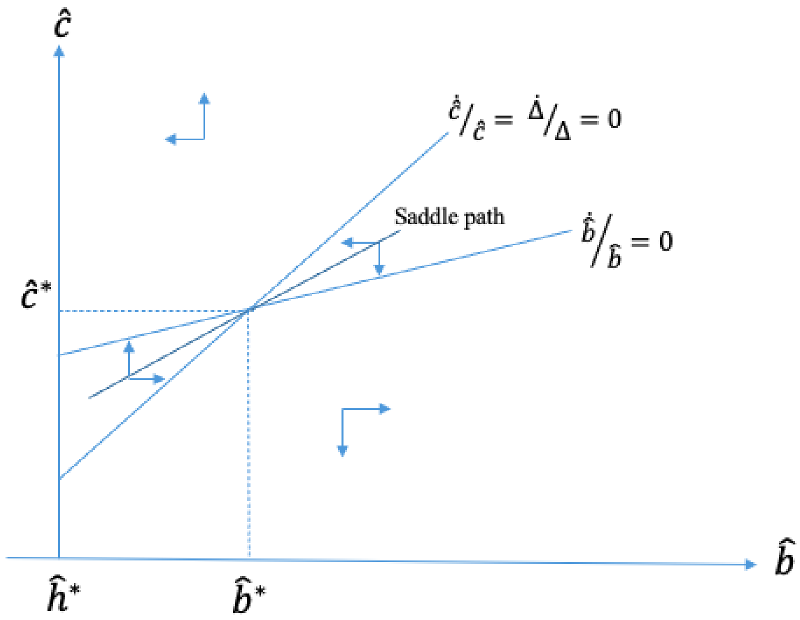

Figure 3: The phase diagram, Case (ii)

[6]Note that the vertical intercepts of the $\frac{\dot{\hat{c}}}{\hat{c}} = \frac{\dot{\Delta}}{\Delta} = 0$ and $\frac{\dot{\hat{b}}}{\hat{b}} = 0$ loci are, respectively, given by: $\frac{\rho+\theta x+\theta(\lambda+n){\Delta^*}^{-1}\hat{h}^*}{r}$ and $\hat{w}^* + (r-n-x)\hat{h}^*$.

4. Numerical results

In this section, by calibrating our model with standard parameter values in the small open economy macroeconomic literature, we first examine whether the development path and the speed of convergence predicted by the model are consistent with their empirical counterparts. We consider the behavior of consumption, physical capital, human capital, all in per effective labor terms; output per capita; growth rate of output per capita; and the saving rate along the transition path from a given initial starting point to the steady state. We then discuss the implications of changes in life expectancy on the speed of convergence and the steady state of the domestic economy. We also use our model to quantify the contribution of life expectancy to the long-run economic performance of sub-Saharan Africa and the OECD countries as well as Canada over the past six decades.

4.1. Calibration

As it is standard in the literature, the benchmark parameter values of our model are chosen on the basis of microeconomic evidence or to match certain long-run averages of the data. Following Barro et al. (1995), the share of physical capital in output is set at $\alpha = 1/3$, the rate of population growth is set at $n = 0.01$, the long-run growth rate of real GDP is set at $x = 0.02$, and the rate of depreciation for physical capital is set at $\delta_K = 0.05$. These values[7] are commonly used in models calibrated to both industrialized as well as emerging small open economies; as in Mendoza (1991), Schmitt-Grohe and Uribe (2003), and García-Cicco et al. (2010), among others.

As mentioned in Alessandrini et al. (2015), regarding the depreciation rate for human capital, there is no agreement in the literature and estimates vary in the range $0.5\% - 5\%$. However, in studies that build a theoretical framework that has a common production function for human capital, physical capital, and consumption, it is normal to set the depreciation rates for both types of capital to a common value. Following this convention, the appropriate depreciation rate for aggregate human capital will be at the high end of the range used in the literature. Therefore, we set δ_h equal to 3.4% so that the aggregate depreciation rate of human capital in our baseline calibration, which includes the loss associated with generation turnover effect (λ), is 5%. Note that

[7] The exception is the rate of depreciation for physical capital, δ_K. In Mendoza (1991) and Schmitt-Grohe and Uribe (2003), it is set at $\delta_K = 0.1$, while in García-Cicco et al. (2010), it is set at $\delta_K = 0.1255$. However, our results are found to be not sensitive to the values chosen for δ_K.

aggregate depreciation rate for human capital is sensitive to the mortality rate and it will change when we study the impact that changes in life expectancy have on the model.

Based on Mendoza (1991) and Schmitt-Grohe and Uribe (2003), who use the value suggested by Kydland and Prescott (1982) and Prescott (1986) for the real interest rate in the U.S. economy, we set the world's real interest rate at $r = 0.04$. This value is also consistent with the estimate by McGrattan and Prescott (2001), who quantify the average return on capital in the noncorporate sector in the U.S. using data from the National Income and Product Accounts (NIPA) by the Bureau of Economic Analysis (BEA) for the period 1990-1999.

Regarding the human capital share parameter in the production function (η), estimates used are typically in the range $1/3 - 2/3$. For example, Mankiw et al. (1992) suggest that a production function that is consistent with their empirical work has equal factor shares of $1/3$ across the three inputs: K, H, L. While their estimate for α is consistent with physical capital's share of income as measured in the National Accounts, their value of η seems low. For example, in the RBC and endogenous growth literature, η is typically set to $2/3$, implying an income share of unimproved labour (L) of zero[8]. Empirical support for using a near zero share of unimproved labor is provided by Gundlach (1995), who re-examines the work of Mankiw et al., but using a more broadly defined measure of human capital. Since human capital accumulation provides an important channel in our model through which changes in life expectancy will impact economic outcomes, the parameter value we select for η is quantitatively important. Therefore, we choose the midpoint $\eta = 1/2$ for our main results, and in Appendix, we do a sensitivity test using values for η near the end points of the range. Additional support for using $\eta = 1/2$ in the main part of the paper, and as our preferred value, is that for this value the speed of convergence in the model is very close to the empirical estimate of 2% per year.

The pure rate of time preference and the inverse of the elasticity of intertemporal substitution parameters are set at $p = 0.01$ and $\theta = 3.5$, respectively. Together these values imply a reasonable steady-state level of domestic saving rate. Values of θ that are less than around 2.7 would imply a counterfactually decreasing transitional behaviour for the domestic saving rate. Finally, the initial

[8] For RBC papers that incorporate human capital investment see Gomme (1993) or Alessandrini et al. (2015), and for endogenous growth papers see Lucas (1988) or Rebelo (1991).

values for the level of technology and human capital per effective labor are set as $Z_0 = 1$ and $\hat{h}(0) = 0.25 \cdot \hat{h}^*$, respectively. Setting $\hat{h}(0) < \hat{h}^*$ allows us to study the transitional dynamics of the model; however, the actual level of $\hat{h}(0)$ is set arbitrarily.

4.2. Transitional dynamics

Panels A, B, and C of Figure 4 below show the time paths of output per capita, economic growth (i.e. growth of output per capita), and the rate of saving in human capital implied by the saddle-path stable solution to the system (30)-(32), respectively, when the hypothetical economy's life expectancy is set at 71[9] (i.e. $\lambda = 1/71$). From panels A and B, we can see that output per capita increases during the transition while the growth of output per capita decreases. These results are consistent with the evidence that the growth rate declines as income increases. Moreover, according to panel C, we see that the saving rate gradually increases during the transition till it reaches its steady state level of 0.316. This result is consistent with the cross country empirical evidence that the saving rate tends to rise to a moderate extent with per capita income during the transition.

The time paths of the small open economy's consumption, human capital, and physical capital per effective labor are shown in panels D, E, and F of Figure 4, respectively. In contrast to the counterfactual conclusions of the baseline open-economy RCK model, in our model, consumption, human capital, and physical capital per effective labor all rise gradually during the transition until they reach their respective positive steady state values of 122.69, 725.36, and 996.26. As mentioned in Section 3.6, the gradual increase in human capital during the transition reflects the feature of diminishing returns to human capital in our model. Moreover, physical capital also rises gradually towards its steady-state value because of the constraint of domestic saving on the accumulation of human capital and the complementarity between the two types of capital in the production function. With the gradual increases in both types of capital, thus, the convergence speeds for capitals and output implied by our model are no longer infinite. The predicted speed of convergence for output per capita is found to be 1.81%, which is close to the empirical estimate of roughly 2% per year.

As in Barro et al. (1995), the speed of convergence implied by our model is found to be not very sensitive to changes in n, x, δ_K, δ_h, p, and θ. However, it is more sensitive to changes in the total

[9] Life expectancy is set to match the life expectancy at birth in Canada in 1960. Later in the section, we focus on the implications of changes in life expectancy for Canada, sub-Saharan African countries, and the OECD countries.

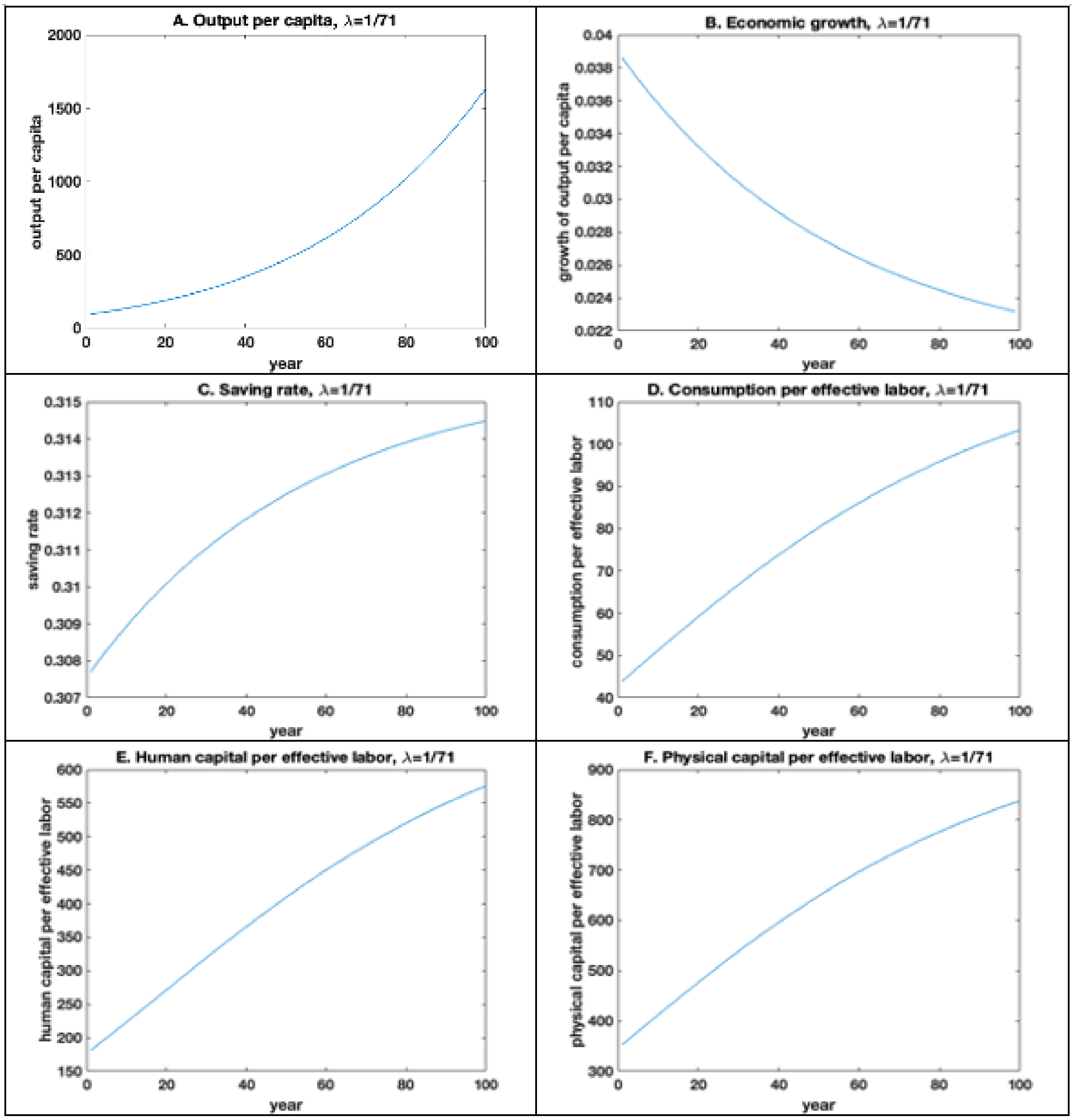

Figure 4: Time paths of the model variables, $\lambda = 1/71$

capital share, $\varepsilon = \frac{\eta}{1-\alpha}$. We find that, as long as the ratio $\frac{\eta}{1-\alpha}$ is kept constant, any changes in α and η have no effect on the speed of convergence. On the other hand, when the values of α and η change in a way that the total capital share falls from its benchmark value of 0.75 to, for example, 0.5—with $\eta = \alpha = 1/3$—the convergence rate jumps from 1.81% to 4.53%.

As discussed in Section 2, in Barro et al. (1995), the world interest rate is preset at $(\rho + \theta x)$—the steady state interest rate that would prevail if the domestic economy were closed—so that consumption and assets of the small open economy converge to positive constants as in the closed economy case, rather than zero and a negative constant. In contrast, in our model, as a result of the finite horizons framework, regardless of the value of the world interest rate, the adjustments in the domestic economy's effective time-preference rate guarantee that the consumption and assets will remain positive. In our calibration, we set $r = 0.04$, which is strictly less than $\rho + \theta x (= 0.08)$; nevertheless, consumption and the both types of capital converge to positive constants.

4.3. Implications of changes in life expectancy

Panel A of Figure 5 below shows the implications of life expectancy on the speed of convergence. We see that the speed of convergence is not very sensitive to changes in life expectancy: a sizable increase in life expectancy leads to only a slight decrease in the implied speed of convergence (it still remains around 2%). However, as can be seen from panel B of Figure 5, an increase in life expectancy does have a notable impact on the steady state level of output per effective labor. For example, when the hypothetical economy's life expectancy increases from 62 to 80, there is a 22.89% increase—from 239.88 to 294.8—in the steady state level of output per effective labor. Given that, according to the latest available data from the World Development Indicators, in 2018, the average life expectancy at birth was around 62 years in sub-Saharan African countries and it was around 80 years in the OECD countries, the aforementioned numerical result can be interpreted as follows: If the average life expectancy at birth in sub-Saharan African countries converges from its current level to the level in their OECD counterparts, there will be a substantial improvement in the region's long-run economic performance—an estimated 22.89% increase in terms of output per effective labor.

In a similar manner, we can also use our model to quantify the contribution of life expectancy

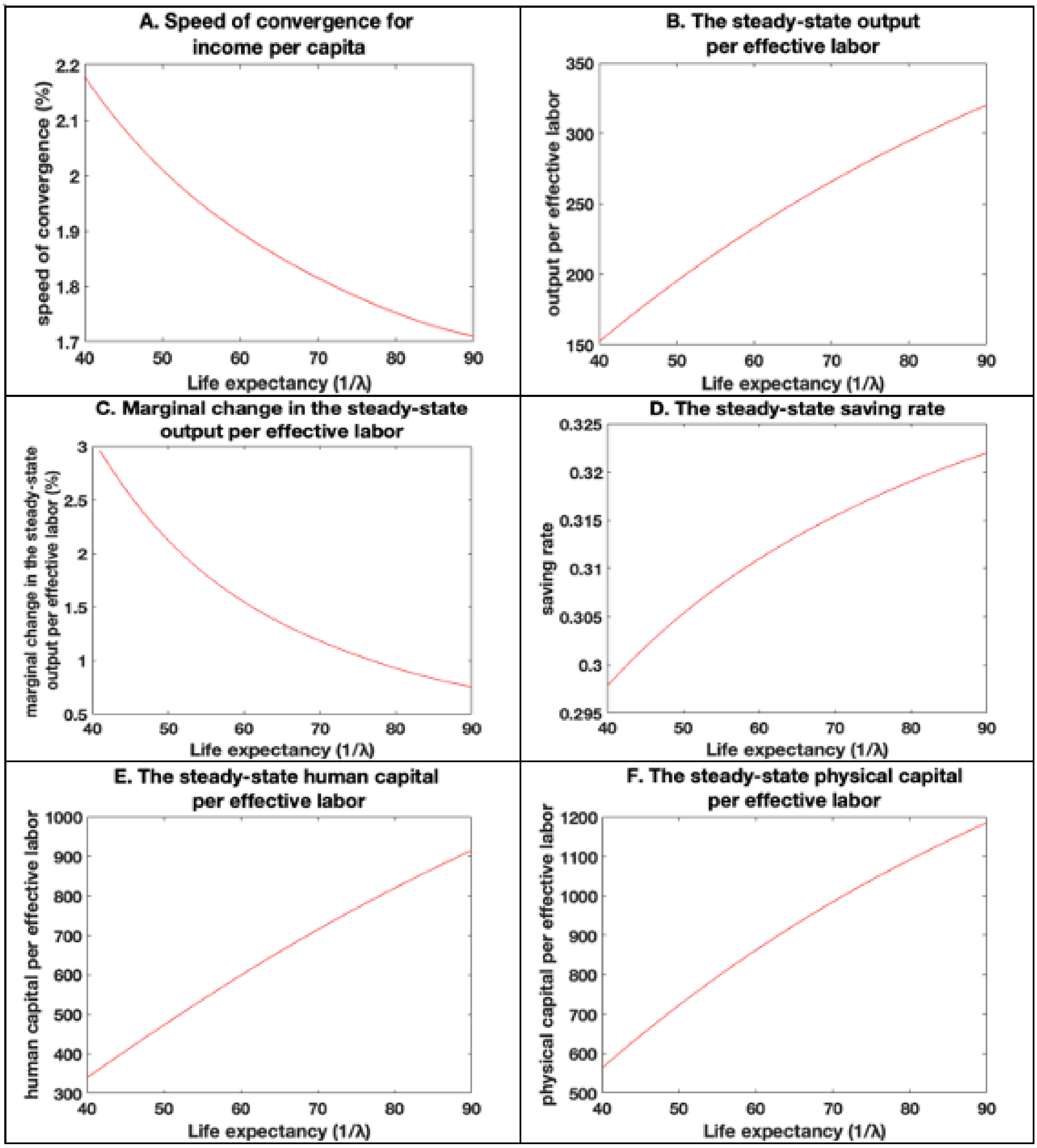

Figure 5: The steady state and life expectancy

to the long-run economic performance of sub-Saharan Africa and the OECD countries as well as Canada over the past six decades. Specifically, between 1960 and 2018, the average life expectancy at birth increased from around 40 years to 62 years—or by 22 years—in sub-Saharan Africa and for the OECD countries the figures are from 67 years to 80 years—or by 13 years. We find that these improvements in life expectancy raised the long-run output per effective labor in sub-Saharan Africa—from 152.31 to 239.88—by an estimated 57.49% and in the OECD region—from 256.49 to 294.8—by an estimated 14.94%, respectively. For Canada, between 1960 and 2018, life expectancy at birth increased from around 71 years to 82 years, which translates to an estimated 11.58% increase in the long-run output per effective labor.

In relation to the quantitative results that we have seen above, our model suggests that long-run output per effective labor is more sensitive to changes in life expectancy at low levels of life expectancy than it is at high levels. This can be seen more clearly from panel C of Figure 5, which illustrates the percentage changes in the steady state output per effective labor as life expectancy increases. We can see that increased life expectancy has positive but diminishing marginal effect on long-run output per effective labor. This result follows from the model's feature of diminishing returns to human capital.

Finally, panels D through F of Figure 5 show how the steady state levels of domestic saving rate in human capital (and thus investment in human capital), human capital, and physical capital increase as life expectancy increases. We note that, in our model, increased life expectancy makes investment in human capital more profitable and thus encourages its accumulation. Given the complementarity between the two types of capital in the production function, the increase in the stock of human capital then leads to an increase in the stock of physical capital. Consequently, there is also a rise in the output level of the domestic economy.

5. Conclusion

We extend the RCK growth model to a small open economy that faces finite horizons and collateral constraints in international borrowing. In our model, the presence of both types of capital and the binding borrowing constraint slow down the infinite speeds of convergence of capital and output. Specifically, our model features diminishing returns with respect to human capital and the transitional dynamics involves a gradual increase in human capital from its initial value to its steady state value. Given the constraint of domestic saving on the accumulation of human capital and the complementarity between the two types of capital in the production function, physical capital and thus output also rise gradually toward their steady-state values. This implies that the convergence speeds for physical capital stock and output will no longer be infinite. Furthermore, given the model's finite-horizons feature, the adjustments in the domestic economy's effective time-preference rate ensure that both consumption and assets remain positive in the long-run.

In our model, increased life expectancy makes investment in human capital more profitable and thus encourages its accumulation. Given the complementarity between the two types of capital in the production function, the increase in the stock of human capital then leads to an increase in physical capital. Consequently, the output of the domestic economy also increases.

Our numerical results suggest that, if the average life expectancy at birth in sub-Saharan African countries converges from its current level to the level in their OECD counterparts, there will be a substantial improvement in the region's long-run economic performance—an estimated 22.89% increase in terms of output per effective labor. We also find that, between 1960 and 2018, improvements in the average life expectancy at birth raised the long-run output per effective labor in sub-Saharan Africa, the OECD region, and Canada by an estimated 57.49%, 14.94%, and 11.58%, respectively.

References

[1] Alessandrini, D., Kosempel, S., and Stengos, T. (2015). The business cycle human capital accumulation nexus and its effect on hours worked volatility. *Journal of Economic Dynamics and Control*, 51:356–377.

[2] Atkeson, A. and Kehoe, P. J. (2000). Paths of development for early- and late-bloomers in a dynamic Heckscher-Ohlin model. Federal Reserve Bank of Minneapolis. Technical report.

[3] Barro, R. and Sala-i Martin, X. (2004). *Economic Growth*. MIT Press.

[4] Barro, R. J., Mankiw, N. G., and Sala-i Martin, X. (1995). Capital Mobility in Neoclassical Models of Growth. *American Economic Review*, 85(1):103–115.

[5] Blackburn, K. and Cipriani, G. P. (2002). A model of longevity, fertility and growth. *Journal of Economic Dynamics and Control*, 26(2):187–204.

[6] Blanchard, O. (1985). Debt, deficits, and finite horizons. *Journal of Political Economy*, 93(2):223–47.

[7] Blankenau, W., Ayhan Kose, M., and Yi, K.-M. (2001). Can world real interest rates explain business cycles in a small open economy? *Journal of Economic Dynamics and Control*, 25(6-7):867–889.

[8] Brueckner, M. and Lederman, D. (2015). Trade openness and economic growth: Panel data evidence from sub-saharan africa. *Economica*, 82:1302–1323.

[9] Cass, D. (1965). Optimum growth in an aggregative model of capital accumulation. *Review of Economic Studies*, 32(3):233–240.

[10] Cervellati, M. and Sunde, U. (2005). Human Capital Formation, Life Expectancy, and the Process of Development. *American Economic Review*, 95(5):1653–1672.

[11] Chakraborty, S. (2004). Endogenous lifetime and economic growth. *Journal of Economic Theory*, 116(1):119–137.

[12] Chang, R., Kaltani, L., and Loayza, N. (2009). Openness can be good for growth: The role of policy complementarities. *Journal of Development Economics*, 90(1):33–49.

[13] De la Croix, D. and Licandro, O. (1999). Life expectancy and endogenous growth. *Economics Letters*, 65(2):255–263.

[14] Dollar, D. and Kraay, A. (2004). Trade, growth, and poverty. *The Economic Journal*, 114(493):F22–F49.

[15] Edwards, S. (1998). Openness, productivity and growth: What do we really know? *The Economic Journal*, 108(447):383–398.

[16] Frankel, J. A. and Romer, D. H. (1999). Does trade cause growth? *American Economic Review*, 89(3):379–399.

[17] Freund, C. and Bolaky, B. (2008). Trade, regulations, and income. *Journal of Development Economics*, 87(2):309–321.

[18] García-Cicco, J., Pancrazi, R., and Uribe, M. (2010). Real business cycles in emerging countries? *American Economic Review*, 100(5):2510–31.

[19] Gomme, P. (1993). Money and growth revisited: Measuring the costs of inflation in an endogenous growth model. *Journal of Monetary Economics*, 32(1):51–77.

[20] Gundlach, E. (1995). The role of human capital in economic growth: New results and alternative interpretations. *Weltwirtschaftliches Archiv*, 131(2):383–402.

[21] Heijdra, B. J. and Ligthart, J. E. (2002). Tax Policy, the Macroeconomy, and Intergenerational Distribution. *IMF Staff Papers*, 49(1):1–6.

[22] Heijdra, B. J. and Romp, W. E. (2009). Human capital formation and macroeconomic performance in an ageing small open economy. *Journal of Economic Dynamics and Control*, 33(3):725–744.

[23] Hu, S.-C. (1999). Economic growth in the perpetual-youth model: Implications of the annuity market and demographics. *Journal of Macroeconomics*, 21(1):107–124.

[24] Kalemli-Ozcan, S. (2002). Does the mortality decline promote economic growth? *Journal of Economic Growth*, 7(4):411–439.

[25] Kalemli-Ozcan, S., Ryder, H. E., and Weil, D. N. (2000). Mortality decline, human capital investment, and economic growth. *Journal of development economics*, 62(1):1–23.

[26] Kawagishi, T. and Mino, K. (2016). Time Preference and Income Convergence in a Dynamic Heckscher–Ohlin Model. *Review of International Economics*, 24(3):592–603.

[27] Koopmans, T. (1965). On the concept of optimal economic growth. *Pontificiae Academiae Scientiarum, The econometric approach to development planning*, pages 225–287.

[28] Kydland, F. E. and Prescott, E. C. (1982). Time to Build and Aggregate Fluctuations. *Econometrica*, 50(6):1345–1370.

[29] Lee, H. Y., Ricci, L., and Rigobon, R. (2004). Once again, is openness good for growth? *Journal of Development Economics*, 75(2):451–472.

[30] Letendre, M.-A. (2004). Capital utilization and habit formation in a small open economy model. *The Canadian Journal of Economics / Revue canadienne d'Economique*, 37(3):721–741.

[31] Letendre, M.-A. and Luo, D. (2007). Investment-specific shocks and external balances in a small open economy model (chocs spécifiques à l'investissement et balances extérieures dans une petite économie ouverte). *The Canadian Journal of Economics / Revue canadienne d'Economique*, 40(2):650–678.

[32] Lucas, R. E. (1988). On the mechanics of economic development. *Journal of monetary economics*, 22(1):3–42.

[33] Mankiw, N. G., Romer, D., and Weil, D. N. (1992). A Contribution to the Empirics of Economic Growth. *The Quarterly Journal of Economics*, 107(2):407–437.

[34] McGrattan, E. and Prescott, E. (2001). Is the stock market overvalued? NBER Working Papers 8077, National Bureau of Economic Research, Inc.

[35] Mendoza, E. (1991). Real business cycles in a small open economy. *American Economic Review*, 81(4):797–818.

[36] Obstfeld, M. (1981). Macroeconomic policy, exchange-rate dynamics, and optimal asset accumulation. *Journal of Political Economy*, 89(6):1142–1161.

[37] Prescott, E. C. (1986). Theory ahead of business-cycle measurement. *Carnegie-Rochester Conference Series on Public Policy*, 25:11–44.

[38] Prettner, K. (2013). Population aging and endogenous economic growth. *Journal of population economics*, 26(2):811–834.

[39] Ramsey, F. P. (1928). A Mathematical Theory of Saving. *The Economic Journal*, 38(152):543–559.

[40] Rebelo, S. (1991). Long-run policy analysis and long-run growth. *Journal of Political Economy*, 99(3):500–521.

[41] Schmitt-Grohe, S. (1998). The international transmission of economic fluctuations:: Effects of U.S. business cycles on the Canadian economy. *Journal of International Economics*, 44(2):257–287.

[42] Schmitt-Grohe, S. and Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, 61(1):163–185.

[43] Uzawa, H. (1968). Time preference, the consumption function, and optimum asset holdings. *Value, Capital, and Growth : Papers in Honour of Sir John Hicks*, pages 485–504.

[44] Yaari, M. E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *The Review of Economic Studies*, 32(2):137–150.

Appendix: Sensitivity Analysis

In the model, increased life expectancy makes an investment in human capital more profitable and thus encourages its accumulation. Under these conditions the elasticity of output with respect to human capital—η from the production function—will be quantitatively important. As mentioned in the model calibration (Section 4.1), values applied for η in the literature are typically in the range $1/3 - 2/3$. In the main text we chose a value of $1/2$ as our preferred estimate, and this was done for two reasons. First, $\eta = 1/2$ is at the midpoint of the range. Second, for $\eta = 1/2$ the model's speed of convergence to the steady state is very close to the value of 2% per year estimated from the empirical growth literature.

In this section, we test the sensitivity of some of our results using values of η near the end points of the range. Specifically, we compare the model's predictions for how life expectancy impacts growth for η is a set of $\{1/3, 1/2, 3/5\}$. Note that η is kept below $2/3$ to avoid having long-run growth become endogenous. Table 2 below summarizes our results. The first row reports the speed of convergence implied by the model. The rows 3-6 show the percentage changes in long-run output due to changes in the average life expectancy at birth. In particular, we report the effects of the following changes in life expectancy in turn: change in the average life expectancy at birth in sub-Saharan African countries between 1960 and 2018; change in the average life expectancy at birth in sub-Saharan Africa if its average life expectancy at birth converges from its 2018 level to the level in their OECD counterparts; change in the average life expectancy at birth in the OECD countries between 1960 and 2018; and change in life expectancy at birth in Canada between 1960 and 2018.

As can be seen from Table 2, compared to the case where $\eta = 1/2$, when $\eta = 1/3$, the speed of convergence for output per effective labor implied by the model jumps from 1.81% to a counterfactually high value of 4.53% per year. Moreover, the changes in life expectancy now lead to much smaller percentage changes in output. On the other hand, when $\eta = 3/5$, the speed of convergence for output plunges to a counterfactually low value of 0.65% per year. Regarding the effects of changes in life expectancy, we can see that the percentage changes in output are now much higher in magnitude than they are for our preferred case of $\eta = 1/2$.

Table 2: Sensitivity analysis: Human capital share, η

		η	
	1/3	1/2	3/5
speed of convergence ($1/\lambda = 71$)	4.53%	1.81%	0.65%
change in life expectancy		% change in output	
40-62	13.81%	57.49%	324.87%
62-80	5.89%	22.89%	94.32%
67-80	3.93%	14.94%	56.7%
71-82	3.07%	11.58%	42.48%