The Environmental Kuznets Curve Under Recycling and Habit Formation

Myrto Kasioumi\textsuperscript{1} and Thanasis Stengos\textsuperscript{2}

\textit{University of Guelph}

\begin{abstract}
Our analysis focuses on a theoretical circular economy model which explains the relationship between pollution and output as well as recycling and output in the context of the Environmental Kuznets Curve (EKC) framework, incorporating in the analysis the habit formation theory. The model is also extended to account for a dynamic relationship between habit stock and the intensity of recycling, as well as to capture the effects of technological progress on both curves. All models find an increasing pattern for the recycling output curve and a downward sloping curve for the EKC, results which are both in agreement with the general patterns of these curves supported by the literature.

\textit{Keywords:} Recycling, Pollution, Economic Growth, Environmental Kuznets Curve (EKC), Circular Economy
\textit{JEL Classification:} C610, O44, Q53
\end{abstract}

\textsuperscript{1}PhD candidate, Department of Economics and Finance, Gordon S. Lang School of Business and Economics, University of Guelph, Guelph, ON N1G 2W1, Canada, mkasoumi@uoguelph.ca

\textsuperscript{2}Professor of Economics, Department of Economics and Finance, Gordon S. Lang School of Business and Economics, University of Guelph, Guelph, ON N1G 2W1, Canada, tstengos@uoguelph.ca

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1. Introduction

The Environmental Kuznets Curve is an important topic of research as it examines how economic growth affects pollution. As environmental degradation worsens, it is important to study how the improvement and the growth of the economy affects the environment. Furthermore, even though there is a rich literature on the EKC, the literature on the relationship between recycling and economic growth is quite limited and most contributions are theoretical in nature and they rely on a circular economy model framework. For example, George et al. [23] found that the EKC does not hold in a closed economy with two factors of production, namely a recyclable input and a polluting resource. Their model suggests that economic growth does not have a positive effect on environmental quality as the EKC would suggest, and as the economy becomes richer, the environmental degradation increase. Kasioumi and Stengos [34] extended George et al. [23] by relaxing its main assumptions, namely that the two inputs are independent and that the recycling input is costless, and reversed its results by confirming the presence of the EKC to be an inverted U curve and the corresponding curve for abatement to be an increasing curve as in Selden and Song [49].

In this context, recycling is a very important tool that can help economies to both reduce the existing pollution level and move towards a more sustainable development (Craighill and Powell [13] (1996), Di Maio et al. [16] (2015), Zhong and Pearce [58] (2018)). In addition, recycling attracts investments and creates employment, so it is also beneficial for the economy (van Beukering et al. [53] (2014)). Handling waste with recycling will not only have a positive socioeconomic and environmental impact, but also positive ecological and energy saving effects (Gubanova et al. [27] (2019)).

Lately, environmental economists are also focusing into habit formation models, as habits can also affect the way we are treating the environment and general environmental quality. Habit formation in economic models is a topic that has found a lot of attention in the past twenty years as it offers a way that may result in different decision patterns. It concerns mainly growth models and more recently it has been added to environmental models too. Löfgren [37] were the first to introduce habit formation in a macroeconomic environmental model. Under their
framework, habit formation is added in the environmental quality, in a model where utility is affected by two consumption goods, one of which has a negative externality to the environment and the environmental quality. Schumacher and Zou [47] introduced habit formation on pollution in an overlapping generations (OLG) model with pollution, which is differentiated by the usual OLG models with pollution since households’ utility is now affected by past pollution levels. That leads to different results not only for the evolution of pollution and capital level over time, but also for the utility level. If generations care more about the pollution level during their lifetime than the next generations’, then the pollution and capital level in the steady state is higher than simpler models without habit and vice versa. Ikefuji [31] used a model where pollution and habit stock of previous consumption levels offers disutility to the agents while consumption offers utility, and tried to analyze how habit formation may change the decisions about pollution abatement activities that households would undertake otherwise, adding technological progress in abatement as well. If individuals care a lot about their habits, the only way to maintain growth is by having a quick technological progress. Once again, we see that habit formation is a key factor that affects the steady state of an economy and which changes the optimal level of the main variables of the models, compared to simpler models which aren’t affected by previous habit stock on either pollution or consumption. That makes habit formation an important aspect that we should take into account when we are studying growth and environmental models.

In the present paper we will try to investigate the relationship between recycling and output as well as the relationship between pollution and output (EKC) using a theoretical model that assumes a circular economy with habit formation on recycling, something that has not been discussed in the literature so far. Our paper constitutes an extension of the theoretical work of Kasioumi and Stengos [34], combining elements of the circular economy model of George et al. [23] with the habit formation framework of Ikefuji [31], in order to investigate the effects that habit formation has on recycling and pollution.

The structure of the paper is as follows: in the next section (Section 2) we offer a brief literature review on habit formation, the EKC, and recycling. In section 3 we discuss the model we are using to analyze the relationship between pollution and output as well as recycling and output under habit formation. That includes the main assumption, functions and the different laws of motion we
used in our solution. Section 4 presents the methodology we based our solution on and Section 5
has our results and plots showing how the relationship between our main variables turned out to
be. Section 6 presents the habit formation and habit intensity nexus and Section 7 introduces
technological progress in the analysis. Finally, we present our concluding remarks on Section 8
which is followed by the Appendix where we discuss a more detailed solution of the three models.

2. Literature review

2.1. Habit formation

Habit refers to a decision or action taken automatically and is inspired by previously taken
activities which offer pleasure or happiness. It is the routine of actions that one subconsciously
undertakes, repeated in a regular basis like brushing one’s teeth (good habit) or smoking (bad
habit). The way that habits are formed is based on four steps: cue, craving, response and reward,
which are followed by our brain in that order every time. Habits can affect our everyday life and
they concern the psychological, economic, health and every other aspect of one’s life. Habit forma-
tion is a topic mainly discussed in psychology which tries to analyze and understand in depth how
habits are formed and how they can affect one’s behaviour and decisions. Lally et al. [35] found
that forming habits vary between different individuals and it can take between 18 to 254 days.
Wood and Rünger [55] stated that forming habits might lead to unwanted consequences as well,
as people start relying on their habits which eventually may lead to stress and addictions through
the reduction of the availability of people to guide their behavior. Sometimes, psychology of habits
can be combined with other fields of study like health and economics and offer interesting insights.
For example, Gregory and Leo [24] used habit formation to test how past attitudes can affect the
future water consumption. They conclude that households with lower habits on consumption of
water tend to be more aware of water conservation issues, which leads to lower water usage in the
future as well. Habit formation is a topic which has also effects on the health field, as molding
someone’s behavior and habits, can lead to beneficial outcomes especially for cases of addiction to
harmful substances. Gardner et al. [22] stated that habit formation is a useful tool which can be
used from doctors for enhancing healthier habits of their patients, sine it is simple to implement
and has unique beneficial long run effects. Later, Gardner and Rebar [21] also showed that the
effect that habit formation has in behavior change mechanism, like interventions, is very promising as it can fortify the new behavior against previous destructive ones.

Habit formation is also used in economic models and specifically it can be used as a characteristic of consumption. Since habits are actions taken automatically, they can affect our decisions which can affect our preferences and our decisions. Some of the most representative papers studying economic growth under a theoretical framework with habit formation are the following. Alvarez-Cuadrado et al. [5] introduced inward looking preferences (habit formation economy) into a macroeconomic model and compared it with the corresponding conventional model (without habit formation). They showed that under this framework, households not only try to maximize their consumption as in any other macroeconomic model, but also the rate that their consumption changes. That however leads to differences in the consumption and savings decisions of the households compared to the simple model, because habits make individuals unable to react to shocks. Chen [11] assumed an endogenous growth model in which households form consumption habits in combination with other already existing habits. This economy ends up having two different balanced growth paths, one with low consumption and high economic growth, and one with the opposite characteristics that is high consumption and low economic growth. However, even though these two steady states are saddle points locally, they cannot be a Pareto equilibrium path. Valente [52] added habit formation in consumption, and negative pollution externalities due to production, on the neoclassic Ramsey growth model and found that the optimal level of consumption and capital is being reduced with the addition of habits.

Habit formation could play a big part in economic theory, as in some cases, it is as powerful as the rationality principle (Becker [7] (1962), Arrow [6] (1986), Hodgson [29] (2004)). However, even though habits can influence our decisions, only people with high environmental attitudes are prone to follow those instincts regularly (Mason et al. [40] (2022)). Another factor that affects the creation of environmentally friendly habits is the cultural background of each person (Hsu and Chen [30] (2021)). Nevertheless, the environmental quality is getting improved under stronger environmental habits (Löfgren [37] (2003)). On the other hand, there is a small part of the literature supporting that habits can also lead to a higher pollution level as well (Zhang [56] (2013), Schumacher and Zou [48] (2006)).
Positive environmental habits can be formed under many ways. For example, businesses can provide psychologically informed incentives that will influence people to adopt eco-friendly behaviors. In addition, governments can add or reduce friction, making it harder for people to continue polluting and easier to start acting more environmentally friendly (Mazar et al. [41] (2020)). Gamification is another way of encouraging the creation of that kind of habits (Hsu and Chen [30] (2021)). Finally, creating green habits at a young age makes the user familiar with the eco-friendly concept and helps maintaining it and following it for many years. Thus, sustainable architecture and a concept of eco-friendly environment at schools, is of high importance to help create and maintain sustainable green habits (Alimin et al. [2] (2021)).

2.2. The Environmental Kuznets Curve

The Environmental Kuznets Curve is an important topic as it examines how economic growth affects pollution. Grossman and Krueger [25] paper was the first to explore the empirical relationship between pollution -specifically air quality- and economic growth. They found that an inverse-U curve can best characterize the relationship between the two previously mentioned variables, with the turning point to be between 4.000 to 5.000 US dollars. Subsequently, Grossman and Krueger [26] using again data for air and water quality for countries from all over the world, examined the relationship between growth and pollution. They confirmed the presence of the EKC to be an inverse-U curve between GDP per capita and pollution with a turning point at an income level less than 8.000$. Esteve and Tamarit [20] also showed that the EKC holds for Spain, using an empirical approach based on a threshold time series model. Cole et al. [12] introduced more environmental indicators than most studies used until then and found that the inverse-U shape of EKC was only supported by local air pollutants, while pollutants of a more global nature have really high turning points with large standard errors. This result is also confirmed by Stern [50]. Later, Roca et al. [46] found that the inverse-U curve should not be generalized neither for all type of emissions nor for global pollutants, but rather for some local ones.

The papers presented so far found evidence that either fully or at least partially supported the EKC following an inverted U pattern. However, there are other studies that refute entirely
the evidence of the inverse-U shape and support instead an N curve pattern. An N-shaped curve was suggested by De Bruyn and Opschoor [14] to be a more appropriate shape that describes the relationship between pollution and economic growth using panel data for European countries. Harbaugh et al. [28] using a more updated version of the dataset on air pollution than that of Grossman and Krueger [26] found that the inverse-U relationship was not supported anymore. Similarly, Dinda [18] also concluded that the EKC is better characterized by an N-shape curve. The literature so far assumed that the initial increase in the pollution is temporary while the decrease of it is permanent. This result is debatable as the decrease of the pollution after a certain level of income per capita may not be permanent, which leads to an N-shape curve for EKC. Most papers have used emission levels as a pollution index even though global environmental degradation is assumed to be a better factor for global EKC. Moreover, although country specific method are more appropriate for the examination of the EKC hypothesis, the most popular analysis used in most of empirical studies was based on cross-sectional data.

The EKC is a very debatable topic in the literature on the environmental economics. There are papers which show that the pollution level decreases with economic growth (Pao et al. [43]), other that find a positive relationship between the variables (Joshi and Beck [32], Luzzati et al. [38]), and other that find an inverse-U pattern for the EKC (Dinda [19], Uchiyama [51], Kasioumi and Stengos [33], Kasioumi and Stengos [34]), and other that find a U shape curve (Begum et al. [8], Ozturk and Al-Mulali [42], Wang et al. [54], Alvarado et al. [4], Demissew Beyene and Kotosz [15]). In addition, the literature supporting an N curve for the EKC continues to grow and it mainly includes recent studies (Bekhet and Othman [9], Li et al. [36], Zhao et al. [57]). Moreover, there are many recent studies showing that the EKC does not have a specific general pattern that holds for all countries, but the shape of it differs according to the country studied (Charfeddine and Mrabet [10], Albulescu et al. [1], Purcel [45]).

2.3. Recycling

The literature on recycling in economics is quite limited and all contributions are theoretical in nature. As we mentioned before, EKC is a topic with great importance as it examines environmental pollution from an economic point of view. However, it is also important to examine if
economic growth has the potential to improve environmental quality. In other words, we should examine if and how economic growth can affect recycling levels which in turn will be beneficial for the environment. Selden and Song [49] using a simple growth model and assuming that the utility of the representative agent is affected positively by consumption and negatively by pollution levels, derived the shape of the EKC to be an inverted U curve and that the corresponding curve for abatement to be a J shape curve. An additional study that dealt with recycling and growth is the paper by Di Vita [17], which showed that economic growth is higher when one introduces materials in production which come from waste recycling, compared to the case that an economy uses only capital, labour and technology to produce its final good. They showed that developed countries increase their welfare by selling recyclable waste to developing countries which can produce secondary materials cheaper. Pittel [44] introduced a new EKC model which varies in two main aspects from the standard EKC model. The model assumes a closed economy which functions under a material balance condition, that is material cannot be destroyed and it can only be converted through recycling. What she found is that EKC might arise during the transition to the long run balanced growth path.

3. The circular economy model with habit formation on recycling

We are following the structure of Kasioumi and Stengos [34], which is based on the theoretical model of George et al. [23], but with some substantial differences on their assumptions. George et al. [23] assumed that the two factors of production are independent, an assumption hard to justify in a real economy, while hence Kasioumi and Stengos [34] assumed that our decision for recycling at each period of time is affected by how much we care about the environment. In other words, the intensity of recycling ($\tau$) is high when we care a lot about the environment and low when we care less about it. Our current model will combine the model of Kasioumi and Stengos [34] and Ikefuji [31], to capture the effects of habit formation on recycling in a closed circular economy. In that way we will approach in a better way the real economy and the way that agents think and take actions, giving a more meaningful interpretation to our results. Our paper is different from other studies, since we incorporate habit formation in recycling, something not done in the literature so
Consider a centralized closed economy with a continuum of identical agents and a benevolent social planner. Without loss of generality, we assume that the number of agents equals one. The agent cares a lot for the state of the environment and so he derives utility from recycling ($R_t$) as well as from consumption ($c_t$). On the other hand, he derives disutility from pollution ($P_t$) since it affects the environment negatively. Last but not least, he derives utility from the comparison of his current recycling level to his habit stock ($h_t$), if the current consumption is higher than the habit stock, and disutility if it is lower. The social welfare function of this economy is given by the following form:

$$U = \int_0^\infty e^{-\rho t} u(c_t, R_t, h_t, P_t) dt$$

(1)

where $u(c_t, R_t, h_t, P_t)$ is the instantaneous utility function and $\rho$ is the rate of time preference. For the first and second order partial derivatives of the above utility function $u$ we assume that: $u_R > 0$, $u_c > 0$, $u_P < 0$, $u_h < 0$, $uPP < 0$, $u_{cc} < 0$, $u_{RR} < 0$, $u_{cP} > 0$, $u_{cR} < 0$ and $u_{ch} > 0$.

The social planner wants to maximize the discounted present value of the future utility stream $u(c_t, R_t, h_t, P_t)$ given by equation (1). To make our analysis tractable we further assume an isoelastic utility function described by the following form:

$$u(c_t, R_t, h_t, P_t) = \left( c_t R_t h_t^{-b} P_t^{-\epsilon} \right)^{1-n} - 1$$

(2)

The first order conditions imply that both recycling ($R$) and consumption ($c$) increase the utility of the consumer, while pollution ($P$) and habit stock ($h$) reduce their utility level. In addition, the second order conditions show that we face diminishing marginal effects for consumption ($u_{cc}$), pollution ($u_{PP}$), and recycling ($u_{RR}$). In other words, every additional unit of pollution reduces the utility level but with a diminishing rate and every additional unit of consumption increases the utility level but with a diminishing rate, something that holds for recycling as well. The assumption that $u_{cP} > 0$ shows that when pollution increases, the utility received from consumption is higher. That can be explained as in order to be able to consume, production should first happen, something that leads to higher pollution levels. In addition, pollution increases from consumption, so higher the above assumption also means that when pollution increases that means that consumption increases as well, something that increases the utility level. Also the assumption of $u_{cR} < 0$, which shows that the utility received from consumption is lower when we increase recycling, can easily be explained as well. In that case, our utility level coming from consumption is lower because now the utility is increased by recycling. That means that consumption and recycling can act as substitute goods. Finally, $u_{ch} > 0$ means that when habit is higher, then recycling is higher, leading to higher production of the recyclable input and higher production of the final good, hence higher consumption.

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where $\epsilon > 0$ is the elasticity of utility with respect to pollution, $n > (\epsilon + 1)/\epsilon$ is the inverse of the elasticity of intertemporal substitution$^4$ and $b \in [0, 1]$ shows the importance of the habit stock. Rewriting the term $Rh^{-b}$ as $R^{1-b}(R/h)^b$, the explanation of $b$ changes to the geometric weight of relative recycling to the habit stock. As $b$ increases, the agent attaches more importance to $R/h$, which means that if the current recycling level is smaller than the past recycling, then the disappointment the agent receives is increased.

The habit stock of recycling can be described as the weighted average of past recycling levels and is given by the following formula:

$$h_t = m \int_{-\infty}^{t} R_s e^{-m(t-s)} ds$$  \hspace{1cm} (3)$$

where $m > 0$ is the discount rate of past recycling levels and $s > 0$ is the total number of $t - 1$ years that we are doing recycling or alternatively, the years of past recycling. The smaller the value of $m$, the higher the degree of persistence of habitual behavior in recycling. In other words, when the agent chooses the current level of recycling, she attaches great importance to her past consumption levels. Differentiating the habit stock formula with respect to time, gives us the law of motion of the habit stock, which is described as:

$$\dot{h} = m(R_t - h_t)$$ \hspace{1cm} (4)$$

In our economy there is a unique final good $q$ which is produced using two factors of production: $x$ which is the recyclable input and $z$ which is the environmentally polluting input. We can assume an input like glass, plastic, or paper for the recyclable input, while the polluting input can be any extracted resource like oil, natural gas, or coal. Both factors of production evolve over time and we also assume that they are substitutes in production$^5$. The production of the final output $q$ is characterized by a concave production function $\phi$, which is affected only by the above two factors.

$^4$According to Alvarez-Cuadrado et al. [5], Alonso-Carrera et al. [3] and Ikefuji [31], $n$ should be greater than one in order to have an interior solution. In our case, the assumption that $\epsilon > 0$ and $n > (\epsilon + 1)/\epsilon$, lead to the fact that $n > 1$ as well, pointing out that our model will have an interior solution.

$^5$That means that there are some marginal cross effects which affect the final result of this model, something ignored by George et al. [23] but included in Kasioumi and Stengos [34].
of production and it is given by the following form:

\[ q_t = \phi(x_t, z_t) \]  

(5)

We assume for convenience and simplicity to have a Cobb-Douglas production function which takes the following form: 
\[ \phi(x_t, z_t) = Ax_t^{(1-\gamma)}z_t^{\gamma} \], where A is a constant technology factor that affects production.

The final good can be used to employ the polluting input or it can be consumed. The final output that is not used, accumulates as waste which can either be recycled or not. The recyclable proportion of the waste stock \( S \) is \( \beta \), while the proportion which cannot be recycled is \( (1-\beta) \).

In addition, the decision on how much to recycle is affected by the importance we attach to the environment and the habit we have of previous recycling, hence there is another parameter that affects the recycling level of each period, \( \tau \), the intensity of recycling. For higher intensity, the agents puts more weight on recycling and the environment, so they increase recycling in order to increase their utility. In that case, the consumption level (the other variable that increases utility) falls, since the agents choose to increase their utility by recycling. On the other hand, for lower \( \tau \) the agents do not care so much about recycling and the environment, so they increase their utility through consumption, and the recycling level falls with time. Thus, the entire recycling level that takes place in each period is equal to \( \tau \beta S \). Taking into account all the previous assumptions, the low of motion of the waste accumulation can be described by the following differential equation:

\[ \dot{S} = \phi(x_t, z_t) - c_t - z_t - \tau \beta S \]  

(6)

Another main assumption of the model is that part of the waste stock is being recycled and is transformed into a useful factor that can be used in the production of the final output. In other words, the recyclable resource \( x \) is equal to the total recycling of the waste stock that takes place each period \( (x_t = R_t = \tau \beta S_t) \). By substituting this into equation 2, 4, 5, and 6 we get the following equations, with the last one to characterize the dynamics of waste accumulation in the economy:
Apart from the waste accumulation path, we also have the pollution accumulation path of this economy. Part of the environmental pollution comes from the use of the polluting resource $z_t$ in the production procedure. Particularly, we assume that each unit of this input leads to $\theta$ units of pollution. Moreover, the pollution level increases by the part of the waste stock that cannot be recycled which generate polluting emissions in a one for one form, that is by $(1 - \beta)S_t$. In addition, the consumption of the final good is producing $\xi$ units of pollution. Last but not least, we assume that the stock of pollution is automatically reduced at each period by a rate $\delta$, since the environment is assumed to be self-renewed. Thus, the low of motion of the pollution accumulation in this economy can be described by the following differential equation:

$$\dot{P} = \theta z_t + (1 - \beta)S_t + \xi c_t - \delta P_t$$

4. The optimal growth in the economy

The social planner wants to maximize the social welfare function (equation 1), subject to the three laws of motion that affect our economy, that is the law of motion of habit stock of recycling, of waste and pollution (equation 8, 10, and 11 respectively). To solve our problem we are using the following Hamiltonian function, taking into account that habit stock ($h$), pollution ($P$) and waste ($S$) are the state variables, while consumption ($c$) and the polluting input ($z$) are the control

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6For example we can assume that the final good that this economy produces is air conditioning, the consumption of which affects the environment negatively.
variables.

\[ H = e^{-\rho t}u(c, \tau \beta S, h, P) + \lambda[\phi(\tau \beta S, z) - c - z - \tau \beta S] \]

\[ + \mu[\theta z + (1 - \beta)S + \xi c - \delta P] + \omega[m(\tau \beta S - h)] \]  

(12)

where \(\lambda, \mu\) and \(\omega\) represent the shadow prices of the constraints we have in our model. Particularly, \(\lambda\) is the shadow price of the waste accumulation path, \(\mu\) is the shadow price of the evolution of pollution and finally \(\omega\) is the shadow price of the low of motion of habit stock. The first order conditions of the above problem are given by the formulas below:

\[ H_c = e^{-\rho t}u_c - \lambda + \mu \xi = 0 \]  

(13)

\[ H_z = \lambda(\phi_z - 1) + \mu \theta = 0 \]  

(14)

which can also be expressed as:

\[ \mu = \lambda \left( \frac{1 - \phi_z}{\theta} \right) \]  

(15)

\[ H_S = \lambda[\tau \beta(\phi_x - 1)] + \mu(1 - \beta) + \omega m \tau \beta + e^{-\rho t} \tau \beta u_x = -\dot{\lambda} \]  

(16)

\[ H_P = e^{-\rho t}u_P - \mu \delta = -\dot{\mu} \]  

(17)

\[ H_h = e^{-\rho t}u_h - m \omega = -\dot{\omega} \]  

(18)

Differentiating equation 13 with respect to time \((t)\) and substituting 13 for \(u_c\) as well as 49 for \(\lambda e^{-(\rho t)}\)\(^\text{7}\), we are able to find the Euler equation for consumption, which is described by the following equation:

\[ \dot{c} = \left( -\frac{u_c}{uc_c} \right) \left( \frac{\theta}{\theta - \xi(1 - \phi_z)} \right) \left[ \frac{\hat{\mu}}{\lambda} - \frac{\hat{\lambda}}{\lambda} + \rho \left( \frac{\mu}{\lambda} \xi - 1 \right) \right] - \frac{u_c P}{uc_c} \dot{P} - \frac{u_c \dot{S}}{uc_c} \tau \beta S - \frac{u_c \dot{h}}{uc_c} \dot{h} \]  

(19)

where \(\left( -\frac{u_c}{uc_c} \right)\) is the intertemporal elasticity of substitution and it is equal to \(1/n\), \(\dot{S}\) and \(\dot{P}\)

\(^7\)A more analytical solution can be found on the Appendix 9 where we present all the equations we used to solve our problem and we also present the final version of the Euler equation for consumption. Hence, equation 48 till 56 are included there as well.
are characterized by equations 10 and 11 respectively, while the terms which include the shadow price of pollution and waste are given in the Appendix 9.

Using the above optimality conditions, we are also able to construct the optimal growth path for the polluting resource. By differentiating equation 14 with respect to time ($t$) and substituting 15 for $\theta$, we get the following equation:

$$
\dot{z} = \left\{ \left( \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mu}}{\mu} \right) (1 - \phi_z) - \tau \beta \dot{S} \phi_z \right\} \frac{1}{\phi_z} \tag{20}
$$

where $\dot{S}$ is characterized by equation 10 and the terms which include the shadow price of waste and pollution are presented in more detail in the Appendix 9, where we also include the growth path of the polluting resource $z$ as well as the steps which we followed to find it.

The solution to the system of the five differential equations described by equations 8, 10, 11, 55, and 56 allows us to find numerical values for our main variables over time, that is for waste ($S$), pollution ($P$), habit stock of recycling ($h$), consumption ($c$) and the polluting factor of production ($z$). Having found them, we are then able to calculate the recycling level ($x = R = \tau \beta S$) and the output level ($q = A(\tau \beta S)^{1-\gamma} z^{\gamma}$) over time, since $\tau, \beta, A$ and $\gamma$ are all parameters for which we can set the values. In the following plots we present our main results, which show the relationship between pollution and output (EKC), recycling and output (EKC with abatement), pollution and consumption, recycling and consumption, as well as consumption and recycling.

5. Results and Discussion

Figure 1 shows the basic EKC which is characterized by an inverse U curve in the literature. In our case, we see that the pollution level falls over time when output increases. Additionally, Figure 2 shows the relationship between recycling and output which is characterized by a J curve in the literature and in our case, it is characterized by an increasing curve which agrees with the

\*Since the system of differential equations we have in our model consists of non linear differential equations, we have to use a computing environment to help us solve it. For that, we used the programming platform MATLAB. In the Appendix 9 we present a table with the parameter choices that generated the graph. The results are robust to various combinations of the chosen values.

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results of Kasioumi and Stengos [34] and Kasioumi and Stengos [33]. We can clearly see that the curve characterizing the EKC is the exact opposite of the one that characterizes the curve between
recycling and output, as EKC is the downward sloping part of an inverse U curve while the recycling output curve is the increasing part of a U curve. That result confirms our intuition as well. In other words, because recycling is a way of pollution abatement it can be considered as the opposite measure of pollution. One reason that this might be the case is the fact that as output increases, economies become richer and they are able to fully cover their basic needs in terms of production, so then they are able to move on and cover some initially secondary needs in terms of recycling.

Under a similar framework but without a habit stock in recycling, Kasioumi and Stengos [34] found that as output increases, pollution increases initially up to a point, after which the pollution starts to fall, in other words confirming the inverse U curve for the EKC. In addition, it was found that the relationship between recycling and output is almost linear and increasing. Our results in this paper are close to the results of Kasioumi and Stengos [34] but with some substantial differences which arise due to the addition of the habit stock in recycling. First of all, we can see that the rate that recycling increases with output is higher in the present paper since the curve is not linear. Moreover, the EKC is now characterized by a downward sloping curve only, in comparison to the inverse U curve of Kasioumi and Stengos [34].

On this point, we want to point out the importance of the parameters $\beta$ and $\tau$ and show how they are related in that specific model. For that reason we provided a sensitivity analysis regarding those two parameters, the results of which are available in the following plots (Figure 3 to Figure 7). A high $\beta$ allows the economy to choose a low $\tau$, which means that as the recyclable proportion of waste increases, we need smaller intensity of recycling to be able to get a combination of lower pollution and higher recycling. In addition, the sensitivity analysis shows that our model produces the same results under different combinations of $\beta$ and $\tau$. 

16
Figure 3: Different $\tau$ under $\beta = 0.05$

Figure 4: Different $\tau$ under $\beta = 0.1$
Figure 5: Different $\tau$ under $\beta = 0.2$.

Figure 6: Different $\tau$ under $\beta = 0.3$. 
6. Habit formation affects the intensity of recycling

We proceed our analysis by incorporating a new important assumption: the habit stock of previous recycling \((h)\) is assumed to affect the intensity of recycling \((\tau)\) in a positive way. We previously assumed that utility can increase with higher levels of recycling and by comparing the stock of habits from previous recycling with the current recycling level. When current recycling is higher than the habit stock utility will rise and the opposite will occur when the habit stock is higher that the current level of recycling. However, as the habit stock increases, the intensity of recycling should increase as well to lead to higher recycling level in the current period compared to the previous ones leading to a higher current utility level. To capture this effect, for simplicity, we assume that the relationship between intensity of recycling and habit stock is linear and it can be described by the following formula:

\[
\tau(h_t) = \eta + rh_t
\]  

(21)
where \( \eta \) is the initial intensity we have, no matter how high or low the habit stock is and it is assumed to be positive, while \( r \) is the coefficient of habit stock which shows how much the intensity is increased when the habit stock is increased by one unit.

Now the intensity of recycling is taken to be a variable in contrast to the basic model where it was just a parameter and as such it will affect most of the previous results. The only function that does not change is the one of the pollution accumulation path since it does neither include the recycling level nor its intensity, thus it is still described by expression 11. The new equations under the new assumption become as follows:

\[
R_t = x_t = \tau(h_t) \beta S_t = (\eta + r h_t) \beta S_t
\]

\[
\phi(x_t, z_t) = \phi(\tau(h_t) \beta S_t, z_t) = A((\eta + r h_t) \beta S)^{(1-\gamma) z_t^\gamma}
\]

\[
u(c_t, \tau(h_t) \beta S_t, h_t, P_t) = \frac{(c_t(\eta + r h_t) \beta S h_t^{-b} P_t^{-c})^{1-n} - 1}{1-n}
\]

\[
\dot{S} = \phi(\tau(h_t) \beta S_t, z_t) - c_t - z_t - \tau(h_t) \beta S_t
\]

\[
= \phi((\eta + r h_t) \beta S_t, z_t) - c_t - z_t - (\eta + r h_t) \beta S_t
\]

\[
\dot{h} = m(\tau(h_t) \beta S_t - h_t) = m((\eta + r h_t) \beta S_t - h_t)
\]

The problem we have to solve is similar to the one we faced in the basic model. Once again, the social planner wants to maximize the social welfare function described by equation 1, subject to the constraint we have for the law of motion of waste (equation 25), of pollution (equation 11), and of habit stock of recycling (equation 26). We will solve the following Hamiltonian function, taking into account that habit stock \( (h_t) \), pollution \( (P_t) \), and waste \( (S_t) \) are the state variables, while consumption \( (c_t) \) and the polluting input \( (z_t) \) are the control variables:

\[
H = e^{-\rho t} u(c_t, \tau(h_t) \beta S_t, h_t, P_t) + \lambda \left[ \phi(\tau(h_t) \beta S_t, z_t) - c_t - z_t - \tau(h_t) \beta S_t \right]
\]

\[
+ \mu [\theta z_t + (1 - \beta) S_t + \xi c_t - \delta P_t] + \omega [m(\tau(h_t) \beta S_t - h_t)]
\]

(27)
where $\lambda$, $\mu$ and $\omega$ have the same interpretation as before and they represent the shadow prices of the three constraints we face in this economy. Particularly, $\lambda$ is the shadow price of the waste accumulation path, $\mu$ is the shadow price of the evolution of pollution, and finally $\omega$ is the shadow price of the low of motion of habit stock. The first order conditions of the above problem are given by the equations below:

\[ H_c = e^{-\rho t}u_c - \lambda + \mu \xi = 0 \]  
\[ H_z = \lambda(\phi - 1) + \mu \theta = 0 \]

which can also be expressed as:

\[ \mu = \lambda \left( \frac{1 - \phi}{\theta} \right) \]  
\[ H_S = \lambda[\tau(h)\beta(\phi - 1)] + \mu(1 - \beta) + \omega m \tau(h) \beta + e^{-\rho t} \tau(h) \beta u_x = -\dot{\lambda} \]  
\[ H_P = e^{-\rho t}u_P - \mu \delta = -\dot{\mu} \]  
\[ H_h = e^{-\rho t}(u_h + u_x \beta S \tau_h) + \lambda \beta S \tau_h(\phi - 1) + m \omega (\beta S \tau_h - 1) = -\dot{\omega} \]

Differentiating equation 28 with respect to time ($t$), and substituting 28 for $u_c$ and 49 for $\lambda e^{\rho t}$, we are able to get the following Euler equation for consumption:

\[ \frac{\dot{c}}{c} = \left( -\frac{u_c}{u_{cc}c} \right) \left( \frac{\theta}{\theta - \xi(1 - \phi)} \right) \left[ \frac{\mu}{\lambda} \xi - \frac{\dot{\lambda}}{\lambda} + \rho \left( \frac{\mu}{\lambda} \xi - 1 \right) \right] - \frac{u_{cP}}{u_{cc}c} \dot{P} \]

\[ - \frac{u_{ch}}{u_{cc}c} \dot{h} - \frac{u_{cx}}{u_{cc}c} \beta \left[ \tau(h) \dot{S} + S \tau_h \dot{h} \right] \]

where $\left( -\frac{u_c}{u_{cc}c} \right)$ is the intertemporal elasticity of substitution and it is equal to $1/n$, $\dot{S}$ and $\dot{P}$ are characterized by equations 25 and 11 respectively, while the terms which include the shadow price of pollution ($\mu$) and waste ($\lambda$) are given in Appendix 10 in more detail.

An important point of departure from the corresponding Euler equation of the basic model described on Section 4, is the last term which includes the intensity of recycling ($\tau$). Previously, $\tau$...
was just a constant parameter, while here it is an important variable which affects our final result and as we can see it appears twice in the equation.

Following similar steps as in our solution for the previous model, we are able to derive the growth path of the environmental polluting resource $z$. Particularly, we differentiate equation 29 with respect to time ($t$) and we substitute equation 30 for $\theta$. That gives us the following equation:

$$
\dot{z} = \left( \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mu}}{\mu} \right) (1 - \phi z) - \beta \phi z x \left( \tau(h) \dot{S} + S \tau \dot{h} \right) \frac{1}{\phi zz} 
$$

(35)

The difference between the current pollution accumulation path and the one described by Section 4, once again comes from the introduction of the intensity of recycling ($\tau$) as a variable. Specifically, the present pollution accumulation path (expression 35) has one more term which includes the law of motion of the habit stock ($\dot{h}$) as well as the waste level ($S$) and the intensity of recycling ($\tau$).

The solution to the system of the differential equations described by equations 11, 25, 26, 60, and 62 allows us to find numerical values for waste ($S$), pollution ($P$), habit stock ($h$), consumption ($c$), and the polluting input ($z$) over time\(^\text{10}\). This allow us to calculate the recycling level ($x = \tau(h) \beta S$) and the output level ($q$) given by equation 23, since all the other unknowns are parameters. The above solution can be seen in the following plots where we present the EKC which describes the relationship between pollution and real GDP, as well as the relationship between recycling and output.

Figure 8 depicts the Environmental Kuznets Curve which shows the relationship between output and pollution\(^\text{11}\). Our results suggest that as economies become richer and the output level increases, the pollution level tends to fall. This curve follows a downward slopping course which matches the curve of the basic model we discussed earlier. Furthermore, Figure 9 shows the recycling-output curve which is characterized by an increasing curve. In other words, as output increases and economies become richer, their recycling level increases as well. We see that our results match the

\(^{10}\)Due to the difficulty level that our differential equation system has, since it consists of non linear equations, we used the programming platform MATLAB to help us solve it. In Appendix 10 we present the choices of parameter values used and show that different parameter choices do not change the main patterns depicted in the figures.

\(^{11}\)The present plot depicts the EKC given high initial output values. This is the reason we see only the downward slopping part of the inverse U shape curve of the EKC. When we set small initial output values, the resulting plot is an increasing curve which is basically the first half part of the inverse U plot of the EKC.
results from the basic model of Section 4.
7. The model with technological progress

In this part of the paper we add one more realistic assumption to the model, which assumes that the production of the polluting input \((z_t)\) is affected by technological progress \((A_t)\). Specifically, we assume a technology that transform the polluting input into a more efficient one, leading to a less polluting production. For that we assume a Harrod-Neutral technology, and specifically one that increases exponentially with time and it is given by:

\[ A_t = A_0 e^{(\alpha t)} \]  
\[ \frac{\dot{A}_t}{A_t} = \alpha \]  

(36)

where \(A_0\) is the initial price of technology and \(\alpha\) shows the rate at which technological progress changes in time. For both \(A_0\) and \(\eta\) we assume that they take positive values only.

The addition of technological progress in the production of the polluting input changes the the production function of the final good, changing in that way some of the main equations and constraints we use in the model as follows:

\[ \phi(x_t, A_t z_t) = \phi(\tau(h_t) \beta S_t, A_t z_t) = \left((\eta + \tau h_t) \beta S\right)^{(1-\gamma)}(A_t z_t)\gamma \]  

(37)

\[ \dot{S} = \phi(\tau(h_t) \beta S_t, A_t z_t) - c_t - z_t - \tau(h_t) \beta S_t \]  

(38)

While the equations for the social welfare function, the instantaneous utility function, the recycling intensity function, the recycling formula, the low of motion of habit, and the pollution accumulation path remain the same as the the previous model, since they are not affected by the production function. That means that those formulas are described by equations 1, 24, 21, 22, 26, and 11 respectively.

12 We can incorporate technological progress in our model in two different ways. The first one assumes that technological progress affects the production of the recyclable input \((x_t)\) and the second one the production of the polluting input \((z_t)\). If we assume the first way, then we need less waste to produce the same amount of the input as before as its production becomes more efficient. That leads to a higher accumulation of waste pile followed by higher pollution. Hence, better technology in the production of the recyclable input leads to lower recycling level and higher pollution level. However, that is not a desirable outcome, which leads us to assume that technological progress affect the production of the polluting input.

13 The polluting input \(z\) will increase faster than the available amount of it, allowing the firm to use less units of it, but producing the same amount of final good \(q\) as before.

14 The addition of technological progress affects only the production function and not the quantity of the polluting input \((z_t)\) directly. That means that even though the quantity of \(z_t\) used is different than the previous cases, this
From the above model we can formulate the following Hamiltonian function to help us solve the social welfare maximization problem:\(^{15}\):

\[
H = e^{-\rho t} u(c_t, \tau(h_t) \beta S_t, h_t, P_t) + \lambda [\phi(\tau(h_t) \beta S_t, A_t z_t) - c_t - z_t - \tau(h_t) \beta S_t] + \mu [c_t + (1 - \beta) S_t + \xi c_t - \delta P_t] + \omega [m(\tau(h_t) \beta S_t - h_t)] \tag{39}
\]

The first order conditions of the above problem are given by the following equations:

\[
H_c = e^{-\rho t} u_c - \lambda + \mu \xi = 0 \tag{40}
\]

\[
H_z = \lambda (A_t \phi_x - 1) + \mu \theta = 0 \tag{41}
\]

which can also be expressed as:

\[
\lambda = \lambda \left( \frac{1 - A_t \phi_x}{\theta} \right) \tag{42}
\]

\[
H_S = \lambda [\tau(h) \beta (\phi_x - 1)] + \mu (1 - \beta) + \omega m \tau(h) \beta + e^{-\rho t} \tau(h) \beta u_x = -\dot{\lambda} \tag{43}
\]

\[
H_P = e^{-\rho t} u_P - \mu \delta = -\dot{\mu} \tag{44}
\]

\[
H_h = e^{-\rho t} (u_h + u_x \beta S \tau_h) + \lambda \beta S \tau_h (\phi_x - 1) + m \omega (\beta S \tau_h - 1) = -\dot{\omega} \tag{45}
\]

Differentiating equation 40 with respect to time (t), and substituting 40 for \(u_c\) and 64 for \(\lambda e^{(\rho t)}\)\(^{16}\), we are able to get the following Euler equation for consumption:

\[
\dot{\frac{c}{c}} = \left( -\frac{u_{ec}}{u_{cc}c} \right) \left( \frac{\theta}{\theta - \xi (1 - A_t \phi_x)} \right) \left[ \frac{\dot{\lambda}}{\lambda} \xi - \frac{\dot{\lambda}}{\lambda} - \rho \left( \frac{\mu}{\lambda} \xi - 1 \right) \right] - \frac{u_{ec} \dot{P}}{u_{cc}c} - \frac{u_{ch}}{u_{cc}c} \beta \left[ \tau(h) \dot{S} + S \tau \dot{h} \right] \tag{46}
\]

---

\(^{15}\)The maximization problem is similar to the previous two cases where the social planner wants to maximize the social welfare function under the waste, pollution, and habit law of motions. Similarly to the previous cases, consumption and the polluting input are the control variables, while habit stock, waste, and pollution are the state variables. Once again, \(\lambda\), \(\mu\) and \(\omega\) have the same interpretation as before and they represent the shadow prices of the waste accumulation path, the evolution of pollution, and finally the low of motion of habit stock respectively.

\(^{16}\)A more detailed solution of this problem can be found on Appendix 11, where we show all the necessary formulas and substitutions, as well as the final version of the Euler equation for consumption.
where $\left(-\frac{w}{u_{cc}}\right)$ is the intertemporal elasticity of substitution and it is equal to $1/n$ as in the previous two cases, $\dot{S}$ and $\dot{P}$ are characterized by equations 38 and 11 respectively, while the terms which include the shadow price of pollution ($\mu$) and waste ($\lambda$) are given in Appendix 11 in more detail.

For the growth path of the environmental polluting resource $z_t$, we differentiate equation 41 with respect to time ($t$) and we substitute equation 42 for $\theta$. That gives us the following equation:

$$
\dot{z} = \left\{ \left( \frac{\lambda}{\mu} - \dot{\mu} \left( 1 - A_t \phi_z \right) - \beta \phi_{zz} \left( \tau(h) \dot{S} + S \tau(h) \dot{h} \right) - \dot{A}(\phi_z + z \phi_{zz}) \right) \right\} \frac{1}{A_t \phi_{zz}} \tag{47}
$$

The difference between the above two equations and the respective ones from the model of Section 5 and Section 6, lies in the introduction of the variable technological progress ($A_t$). Here, both equations include that variable in many terms, especially the pollution accumulation path where we also see the appearance of a new term affected by the change of the technology in time.

The solution to the system of the differential equations described by equations 11, 26, 38, 70, and 71 allows us to find numerical values for the important variables describing the model ($S$, $P$, $h$, $c$, and $z$) over time\footnote{We used the programming platform MATLAB to help us solve the system of the five differential equations, as they are non linear functions unable to be solved otherwise. Appendix 11 includes a table with the choices of the parameter values while it also shows that different parameter choices do not change the relationship between the variables.}, which also allows us to calculate the recycling level and the output level.

The results of this model are presented in Figure 10 and 11:
Under technological progress on the production of the polluting input the relationship between
pollution and output on one hand and recycling and output on the other hand, remain similar to the previous two cases. That means that the Environmental Kuznets Curve is characterized by a decreasing curve and recycling is increasing with output.

8. Conclusion

In an effort to model an economy with more realistic characteristic, we included a variable that describe people and their way of thinking in a more accurate way, which is habit formation. Habits are regularly repeated routines of behaviours that people do subconsciously. They affect decisions and preferences and for those reasons habit formation has been introduced in many economic models.

Our analysis describes a closed circular economy in which there is one final good that is produced by a recyclable and a polluting input. Agents receive utility from consumption and recycling while they receive disutility from pollution. Additionally, they have a habit stock of previous recycling levels which offers them higher utility when their current consumption is higher than their habit stock. Our model combines the closed circular economy model with recycling of Kasioumi and Stengos [34] and the habit formation theory of Ikefuji [31]. However, we offer a new framework, since we include habit formation on recycling and not consumption which is the usual. In addition, we extent the analysis by incorporating two more models including more realistic assumptions. Specifically, the first one assumes that the recycling level of each period is affected by the preferences of the representative agent (intensity of recycling), which is influenced by their habits for recycling, while the second one assumes that the production function is also influenced by technological progress.

Our main purpose with this study is to investigate the relationship between pollution and output (EKC) as well as recycling and output, in addition to a comparison between the results of this study and the one of Kasioumi and Stengos [34]. We find that the EKC is characterized by a downward sloping curve while the recycling-output curve by an increasing curve which confirms our expectations. Recycling is a mean of environmental abatement so it can be considered as the opposite variable of pollution, which leads to the opposite curve of the EKC we found here. Many
papers in the literature find an increasing or even a J curve for the plot of recycling and output (see Kasioumi and Stengos [34], Kasioumi and Stengos [33], Selden and Song [49]) and an inverse U curve for the EKC (see Grossman and Krueger [26], Grossman and Krueger [25], Dinda [18] for specific air pollutants). Kasioumi and Stengos [34] results agree with the general pattern of the literature for the two curves. They found an inverse U curve for the EKC but an increasing, almost linear, curve for the recycling-output plot. This study offers robustness to the results of Kasioumi and Stengos [34] since the general pattern for the two curves still holds. However, in our case both of the curves decrease and increase in a higher rate than the ones of Kasioumi and Stengos [34], due to the addition of the habit stock in recycling. In addition, our results contradict the literature which supports that these curves do not exist\textsuperscript{18}, while they agree and offer robustness to the literature which support the existence of the curves.

Our results can be interpreted by the Maslow’s Hierarchy of Needs (Maslow [39]), where they explain that everyone have 5 levels of needs that want to satisfy, starting with the most basic ones (like having food and a house), moving on to the more self actualization needs (like having creative activities). Each person can try to achieve the next level of needs only when they have already fulfilled the previous one. All these hold for every single person, so they can also be generalized for the total society and economy as well. Hence, in our case, as output increases and allows the economies to become richer, the basic levels of needs as having production and growth are satisfied, which then allows the economies to move on to the next levels and start achieving some more self fulfillment needs, as recycling.

Appendix

9. Solution of basic model

In that section we will provide some further information on the solution of the model described in Section 3, as well as some insight on the way we were able to find the Euler equation for

\textsuperscript{18}Some of the papers supporting an N shape curve for the EKC or the non existence of the curve are: De Bruyn and Opschoor [14], Dinda [18], Harbaugh et al. [28]. The literature studying the relationship between recycling and output is comparatively new and does not consist of many papers, however George et al. [23] under a theoretical model found that it does not hold.
consumption \((c)\) and the polluting resource \((z)\). We will show analytically the steps we followed in addition to all formulas used to help us find the differential equations we used to solve our model. From equations 13 and 14, we are able to find \(\lambda\) and \(\mu\) as well as \(e^{-\rho t}u_c\theta\) and by combining them with equations 16 and 17, we are getting back the following equations which helped us determine the Euler equations:

\[
\lambda = \frac{e^{-\rho t}u_c\theta}{\theta - \xi(1 - \phi_z)} \tag{48}
\]

For our solution it is also convenient if we rewrite the above expression as follows:

\[
e^{\rho t}\lambda = \frac{u_c\theta}{\theta - \xi(1 - \phi_z)} \tag{49}
\]

\[
\mu = \frac{e^{-\rho t}u_c(1 - \phi_z)}{\theta - \xi(1 - \phi_z)} \tag{50}
\]

\[-\frac{\dot{\lambda}}{\lambda} = \tau\beta(\phi_x - 1) + (1 - \beta)\frac{\mu}{\lambda} + \frac{\tau\beta m\omega[\theta - \xi(1 - \phi_z)]}{e^{-\rho t}u_c\theta} + \frac{\tau\beta u_x[\theta - \xi(1 - \phi_z)]}{u_c\theta} \tag{51}\]

\[
\frac{\dot{\mu}}{\mu} = \frac{u_P[\theta - \xi(1 - \phi_z)]}{u_c(1 - \phi_z)} - \delta \tag{52}
\]

\[
-\frac{\dot{\mu}}{\mu} = \frac{u_P[\theta - \xi(1 - \phi_z)]}{u_c(1 - \phi_z)} - \delta \tag{53}
\]

\[
\frac{\mu}{\lambda} = \frac{1 - \phi_z}{\theta} \tag{54}
\]

By doing all the necessary substitution from the above equations, to equation 19, we find the final version of the Euler equation for consumption which is given by:

\[
\frac{\dot{c}}{c} = \left(\frac{1}{n}\right) \left(\frac{\theta}{\theta - \xi(1 - \phi_z)}\right) \left[\tau\beta(\phi_x - 1) + \frac{1}{\theta}(1 - \phi_z)(1 - \beta + \xi(\delta + \rho)) - \rho\right] + \left(\frac{1}{u_c c}\right) \left[\xi U_P - \tau\beta U_x - e^{\rho t}wm\tau\beta - U_c P[\theta z + (1 - \beta)S + \xi c - \delta P] - \tau\beta U_{cx}\phi(\tau\beta S, z) - c - z - \tau\beta S\right] \tag{55}\]

Based on the above equation, we can see that the time path of consumption is affected by the variables: pollution \((P)\), production \((\phi)\), consumption \((c)\), polluting input \((z)\) and waste \((S)\),
as well as the marginal product of production with respect to recycling and polluting input, the marginal utility with respect to consumption and the polluting input. In addition, equation 55 is affected by the parameters of the model, that is by the rate at which pollution is created by using the polluting input in the production procedure (\(\theta\)), the rate at which the consumption of the final good produces pollution units (\(\xi\)), the intensity of recycling (\(\tau\)), the proportion of the waste stock that can be recycled (\(\beta\)), the rate in which the pollution decays naturally (\(\delta\)), the time preference parameter (\(\rho\)), the shadow price of habit stock (\(w\)), the discount rate of past recycling (\(m\)) and the elasticity of intertemporal substitution (1/n).

Here, we are also able to show a more analytical version of the optimal growth path of the polluting resource. By substituting 15 for \(\mu\), 16 and 17 for \(\dot{\lambda}\) and \(\dot{\mu}\) respectively, 10 for \(\dot{S}\) and 48 for \(\lambda\), on equation 20, we are able to get the following more complicated equation for the growth path of the polluting resource:

\[
\dot{z} = \left(\frac{\phi_z - 1}{\phi_{zz}}\right) \left\{ \delta - \frac{u_P\theta}{u_c(1 - \phi_z)} + \frac{U_P\xi}{U_c} + \tau \beta \left[ \phi_x - 1 + \frac{wm(\theta - \xi(1 - \phi_z))}{e^{-\rho U_c\theta}} \right] + \frac{U_x(\theta - \xi(1 - \phi_z))}{U_c\theta} \right\} - \frac{\tau \beta \phi_{xx}}{\phi_{zz}} [\phi(\tau S, z) - z - e - \tau S]
\]

The low of motion of the polluting resource \(\dot{z}\) (equation 56), is affected by similar variables and parameters as the time path of consumption (equation 28). Particularly, the marginal product of production with respect to the polluting resource and the recycling as well as the marginal utility with respect to the polluting input, the recycling level and the consumption, all affect \(\dot{z}\). Furthermore, the polluting input (\(z\)), the consumption (\(c\)) and the waste (\(S\)), also affect \(\dot{z}\). Finally, the following parameters affect our formula too: the rate in which the pollution decays naturally (\(\delta\)), the rate at which pollution is created by using the polluting input in the production procedure (\(\theta\)), the rate at which the consumption of the final good produces pollution units (\(\xi\)), the intensity of recycling (\(\tau\)), the proportion of the waste stock that can be recycled (\(\beta\)), the shadow price of habit stock (\(w\)) and the discount rate of past recycling (\(m\)).

In the following table (Table 1) we present the values of the parameters we used to generate the plots of the initial model. However, our results are robust to various combinations of the parameters.
values.

Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>technological progress</td>
<td>2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>recyclable proportion of waste</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>weight of the polluting resource in the production function</td>
<td>0.3</td>
</tr>
<tr>
<td>$\delta$</td>
<td>regeneration rate of environment</td>
<td>0.3</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>elasticity of utility with respect to pollution</td>
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</tr>
<tr>
<td>$\theta$</td>
<td>pollution units created by the polluting resource</td>
<td>1</td>
</tr>
<tr>
<td>$\xi$</td>
<td>pollution units created by consumption</td>
<td>2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>time preference</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau$</td>
<td>intensity of recycling</td>
<td>4</td>
</tr>
<tr>
<td>$b$</td>
<td>importance of habit stock in utility function</td>
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</tr>
<tr>
<td>$m$</td>
<td>discount rate of past recycling levels</td>
<td>0.5</td>
</tr>
<tr>
<td>$n$</td>
<td>inverse of the elasticity of intertemporal substitution</td>
<td>2</td>
</tr>
<tr>
<td>$w$</td>
<td>shadow price of habit stock</td>
<td>1</td>
</tr>
</tbody>
</table>

10. Solution of extension model 1

In that section, we will provide some more information about the methodology we used to solve for the Euler equation for consumption, and the accumulation path of the polluting resource of the second model presented above (Section 6). As we can easily notice, equation 28 is the same as equation 13 and equation 29 is the same as equation 14 of our basic model. That leads us to have the same formulas for the shadow price of waste ($\lambda$) and pollution ($\mu$), which are described in Appendix 9 in equations 48-50. Using those formulas, we are able to construct all the fractions we have to substitute in equation 34 to find the final version of the Euler function for consumption. In other words, we find:

$$\frac{\mu}{\lambda} = 1 - \frac{\phi_z}{\theta}$$ (57)
\[ -\frac{\dot{\lambda}}{\lambda} = \tau(h)\beta(\phi_x - 1) + (1 - \beta)(1 - \phi_z) \frac{1}{\theta} + \frac{\tau(h)\beta m\omega[\theta - \xi(1 - \phi_z)]}{e^{-\rho t}u_c\theta} + \frac{\tau(h)\beta u_x[\theta - \xi(1 - \phi_z)]}{u_c\theta} \] (58)

\[ \frac{\dot{\mu}}{\lambda} = \frac{(1 - \phi_z)\delta}{\theta} - \frac{u_P[\theta - \xi(1 - \phi_z)]}{u_c\theta} \] (59)

and by doing all the necessary substitution from the above equations and equations 25 for \( \dot{S} \), 26 for \( \dot{h} \), and 11 for \( \dot{P} \) to equation 34, we find the final version of the Euler equation for consumption which is described by the following function:

\[ \frac{\dot{c}}{c} = \left( \frac{1}{n} \right) \left( \frac{\theta}{\theta - \xi(1 - \phi_z)} \right) \left[ \tau(h)\beta(\phi_x - 1) + \frac{1}{\theta}(1 - \phi_z)(1 - \beta + \xi(\delta + \rho)) - \rho \right] 
+ \left( \frac{1}{u_c\rho} \right) \left[ \xi U P - \tau(h)\beta U_x - e^{\rho t}wm\tau(h)\beta - U_{cp}[\theta z + (1 - \beta)S + \xi c - \delta P] 
- U_{ch}(\tau S - h) - \beta U_{cx}[\tau(h)(\phi(\tau S, z) - c - z - \tau \beta S) + S\tau h m(\tau(h)\beta S - h)] \right] \] (60)

To find the final version of the pollution accumulation path, we have to first find the formula that characterize the \( \frac{\dot{\mu}}{\mu} \) which is part of equation 35. As previously, we will combine the formula of \( \mu \) describes by equation 50 and the \( \dot{\mu} \) given by equation 32, which gives us the following:

\[ -\frac{\dot{\mu}}{\mu} = \frac{u_P[\theta - \xi(\alpha - \phi_z)]}{u_c(\alpha - \phi_z)} - \delta \] (61)

As we can see, the above formula is the same as the corresponding formula of the basic model. That holds because the addition of the intensity of recycling \( \tau \) as a variable to our model, does not affect neither the function of \( \mu \) nor the one of \( \dot{\mu} \). Substituting equation 58 for \( \frac{\dot{\lambda}}{\lambda} \), equation 61 for \( \frac{\dot{\mu}}{\mu} \) as well as equation equation 25 for \( \dot{S} \) and 26 for \( \dot{h} \), we are able to derive the final version of
the pollution accumulation path, given by the following formula:

\[
\dot{z} = \left( \frac{\phi_z - 1}{\phi_{zz}} \right) \left\{ \delta - \frac{u_\rho \theta}{u_c (1 - \phi_z)} + \frac{U_p \xi}{U_c} + \tau(h) \beta \left[ \phi_x - 1 + \frac{wm(\theta - \xi(1 - \phi_z))}{e^{-\rho t} U_c \theta} \right] \\
+ \frac{U_x (\theta - \xi(1 - \phi_z))}{U_c \theta} + \frac{(1 - \beta)(1 - \phi_z)}{\theta} \right\} \\
- \beta \frac{\phi_{zz}}{\phi_z} \left[ \tau(h) \left[ \phi(\tau(h) \beta S, z) - z - \tau(h) \beta S \right] + S \tau h \left( \tau(h) \beta S - h \right) \right] 
\] (62)

We see that both accumulation paths for the extension model are similar to the ones we got from the initial model but more complicated as we have included in our analysis a new variable, the intensity of recycling \((\tau)\) which is affected positively by the habit stock \((h)\). Hence, the functions here depend from the same variables and parameters as the two accumulation paths of the basic model, in addition to some new. Specifically, the accumulation path of consumption is affected by the intensity of recycling \((\tau)\) which is a variable now instead of a parameter, as well as the marginal product of the intensity of recycling with respect to the habit stock. Similarly, the time path of the polluting resource is additionally affected by the intensity of recycling \((\tau)\).

In the following table (Table 2) we present the values of the parameters we used to generate the plots of the extension model described in Section 6. Even though we chose the following specific values, our results are robust to many other combinations of the parameters values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>technological progress</td>
<td>2</td>
</tr>
<tr>
<td>(\beta)</td>
<td>recyclable proportion of waste</td>
<td>0.5</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>weight of the polluting resource in the production function</td>
<td>0.3</td>
</tr>
<tr>
<td>(\delta)</td>
<td>regeneration rate of environment</td>
<td>0.3</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>elasticity of utility with respect to pollution</td>
<td>1</td>
</tr>
<tr>
<td>(\eta)</td>
<td>initial intensity level</td>
<td>40</td>
</tr>
<tr>
<td>(\theta)</td>
<td>pollution units created by the polluting resource</td>
<td>10</td>
</tr>
<tr>
<td>(\xi)</td>
<td>pollution units created by consumption</td>
<td>2</td>
</tr>
<tr>
<td>(\rho)</td>
<td>time preference</td>
<td>0.5</td>
</tr>
<tr>
<td>(\tau)</td>
<td>intensity of recycling</td>
<td>18</td>
</tr>
<tr>
<td>(b)</td>
<td>importance of habit stock in utility function</td>
<td>0.5</td>
</tr>
<tr>
<td>(m)</td>
<td>discount rate of past recycling levels</td>
<td>0.5</td>
</tr>
<tr>
<td>(n)</td>
<td>inverse of the elasticity of intertemporal substitution</td>
<td>2</td>
</tr>
<tr>
<td>(r)</td>
<td>coefficient of habit stock</td>
<td>0.5</td>
</tr>
<tr>
<td>(w)</td>
<td>shadow price of habit stock</td>
<td>1</td>
</tr>
</tbody>
</table>
11. Solution of extension model 2

We provide the detailed steps we used to solve the third extension model and specifically the methodology to find both the new Euler equation for consumption and the pollution accumulation path of Section 7. We followed the same methodology as in the previous two cases and we found that the formulas for the shadow price of waste ($\lambda$) and pollution ($\mu$) are given by:

$$\lambda = \frac{e^{-\rho t}u_c\theta}{\theta - \xi(1 - A_t\phi_z)} \quad (63)$$

Once again, for the purpose of our analysis, it is convenient to rewrite the above expression as follows:

$$e^{\rho t}\lambda = \frac{u_c\theta}{\theta - \xi(1 - A_t\phi_z)} \quad (64)$$
$$\mu = \frac{e^{-\rho t}u_c(1 - A_t\phi_z)}{\theta - \xi(1 - A_t\phi_z)} \quad (65)$$

Using those formulas, we are able to construct all the fractions we need to substitute in equation 46 to find the final version of the Euler function for consumption and equation 47 for the pollution accumulation path. In other words, we find:

$$\frac{\dot{\mu}}{\lambda} = \frac{\mu}{\lambda} \delta - \frac{u_P[\theta - \xi(1 - A_t\phi_z)]}{u_c\theta} \quad (66)$$

$$-\frac{\dot{\lambda}}{\lambda} = \tau(h_t)\beta(A_t\phi_x - 1) + (1 - \beta)\frac{\dot{\mu}}{\lambda} + \frac{\tau(h_t)\beta m\omega[\theta - \xi(1 - A_t\phi_z)]}{e^{-\rho t}u_c\theta} + \frac{\tau(h_t)\beta u_x[\theta - \xi(1 - A_t\phi_z)]}{u_c\theta} \quad (67)$$

$$-\frac{\dot{\mu}}{\mu} = \frac{u_P[\theta - \xi(1 - \phi_z)]}{u_c(1 - \phi_z)} - \delta \quad (68)$$
$$\frac{\mu}{\lambda} = \frac{1 - A_t\phi_z}{\theta} \quad (69)$$

and by doing all the necessary substitution from the above equations and equations 38 for $\dot{S}$, 26
for $\dot{h}$, and 11 for $\dot{P}$ to equation 46, we find the final version of the Euler equation for consumption given by:

$$\begin{align*}
\dot{c} &= \left(\frac{1}{n}\right) \left(\frac{\theta}{\theta - \xi(1 - A_t \phi_z)}\right) \left[\tau(h_t)\beta(\phi_x - 1) + \frac{1}{\theta}(1 - A_t \phi_z)(1 - \beta + \xi(\delta + \rho)) - \rho\right] \\
&\quad + \left(\frac{1}{u_x c}e\right) \left[\xi U_P - \tau(h_t)\beta U_x - e^{\rho t}m \beta \tau(h_t) - U_c \theta z_t + (1 - \beta) S_t + \xi c_t - \delta P_t\right] \\
&\quad - U_{cx} m(\tau(h_t)\beta S - h_t) - \beta U_x \left[\tau(h_t)\left(\phi(\tau(h_t)\beta S_t, A_t z_t) - c_t - z_t - \tau(h_t)\beta S_t\right)
\right] \\
&\quad \quad + S \tau_h m(\tau(h_t)\beta S_t - h_t) \right] (70)
\end{align*}$$

To find the final version of the pollution accumulation path, we substitute equation 67 for $\dot{\Lambda}$, equation 68 for $\dot{\mu}$, equation 38 for $\dot{S}$, and 26 for $\dot{h}$:

$$\begin{align*}
\dot{z} &= \left(\frac{A_t \phi_z - 1}{A_t \phi_{zz}}\right) \left\{\delta - \frac{u_p \theta}{U_x (1 - A_t \phi_z)} + \frac{U_p \xi}{U_x} + \tau(h_t)\beta \left[\phi_x - 1 + \frac{wm(\theta - \xi(1 - A_t \phi_z))}{e^{-\rho t} U_x \theta}\right]
\right\} \\
&\quad + \frac{U_x \left(\theta - \xi(1 - A_t \phi_z))\right)}{U_x \theta} + \left(1 - \beta\right)(1 - A_t \phi_z) - \alpha \left[\frac{\phi_z}{\phi_{zz}} - z\right] \\
&\quad - \frac{\beta \phi_{xx}}{\phi_{zz}} \left[\tau(h_t)\left(\phi(\tau(h_t)\beta S_t, A_t z_t) - z_t - c_t - \tau(h_t)\beta S_t\right) + S \tau_h m(\tau(h_t)\beta S_t - h_t)\right] (71)
\end{align*}$$

We see that both accumulation paths for the extension model are similar to the ones we got from the previous two models but are more complicated as they include the variable technological progress ($A_t$) which affects directly the production function. Hence, the functions here depend from the same variables and parameters as the two accumulation paths of the basic model and extension model one, in addition to some $A_t$.

In the following table (Table 3) we present the values of the parameters we used to generate the plots of the extension model described in Section 7. Even though we chose the following specific values, our results are robust to many other combinations of the parameters values.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>technological progress</td>
<td>2</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>increase rate of technology</td>
<td>0.3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>recyclable proportion of waste</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>weight of the polluting resource in the production function</td>
<td>0.3</td>
</tr>
<tr>
<td>$\delta$</td>
<td>regeneration rate of environment</td>
<td>0.3</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>elasticity of utility with respect to pollution</td>
<td>1</td>
</tr>
<tr>
<td>$\eta$</td>
<td>initial intensity level</td>
<td>27</td>
</tr>
<tr>
<td>$\theta$</td>
<td>pollution units created by the polluting resource</td>
<td>1</td>
</tr>
<tr>
<td>$\xi$</td>
<td>pollution units created by consumption</td>
<td>2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>time preference</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau$</td>
<td>intensity of recycling</td>
<td>1</td>
</tr>
<tr>
<td>$b$</td>
<td>importance of habit stock in utility function</td>
<td>0.5</td>
</tr>
<tr>
<td>$m$</td>
<td>discount rate of past recycling levels</td>
<td>0.5</td>
</tr>
<tr>
<td>$n$</td>
<td>inverse of the elasticity of intertemporal substitution</td>
<td>2</td>
</tr>
<tr>
<td>$r$</td>
<td>coefficient of habit stock</td>
<td>0.5</td>
</tr>
<tr>
<td>$w$</td>
<td>shadow price of habit stock</td>
<td>1</td>
</tr>
</tbody>
</table>
References


[51] Katsuhisa Uchiyama. Environmental kuznets curve hypothesis. In *Environmental Kuznets curve hypothesis and


